Ideas and Firm Dynamics When It Takes Two to Tango

Seula Kim* Jane Olmstead-Rumsey† Honghao Wang‡

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Abstract

Teams of inventors, rather than inventors working alone, now produce most (80%) of U.S. patents, compared to just 45% in 1976. Over the same period, employment concentration of inventors at large firms has risen six times faster than overall employment concentration. To explain these patterns, we build a model of team formation within the firm and embed it in a general equilibrium model of the inventor labor market. An increase in the relative productivity of teams compared to solo inventors raises firms' returns to scale in inventor labor by exploiting collaborations across employees. Estimating this model on the patenting output and inventor employment of all patenting firms in the U.S. reveals that the rise in team productivity can explain the rise inventor employment concentration. Knowledge spillovers across firms, which are important in the innovation context, imply that the allocation of inventors to firms is not necessarily efficient. In the presence of spillovers, the rise of teams exacerbates misallocation by reducing the size of small firms that tend to produce high knowledge spillovers.

Keywords: innovation, R&D team, inventor allocation, firm dynamics, economic growth

*Pennsylvania State University; Email: seulakim@psu.edu

†LSE and CEPR; Email: j.olmstead-rumsey@lse.ac.uk

‡Northwestern University; Email: honghao.wang@kellogg.northwestern.edu. We thank Gadi Barlevy, Francisco Brahm, Santiago Caicedo, Maarten De Ridder, Nik Engbom, Lukas Freund, Seb Graves, Ben Moll, Chris Moser, Ezra Oberfield, Jeremy Pearce, Ben Pugsley, and Chris Tonetti for helpful comments and discussions. We thank Dongjae Lee, Francisco Libano-Monteiro and Shiori Tanaka for outstanding research assistance.

1 Introduction

The development of new ideas is a key driver of economic growth. Ideas are produced either by individual inventors or by collaborations among teams of inventors. Teams have become increasingly important in idea production in recent decades (Jones 2009; Pearce 2023), both in patenting and in scientific research. Existing models of inventor team formation study important dimensions of collaboration—such as specialization, sorting, human capital accumulation, and declining communication costs—but abstract from firm boundaries.

The central premise of this paper is that firm boundaries impose meaningful constraints on inventor collaboration. Firms shape how inventors are matched and organized into collaborative units and therefore play a fundamental role in the rise of knowledge teams. In this paper, we study how the idea production function interacts with firm boundaries and how this interaction shapes firm outcomes and the allocation of inventors across firms. We first document recent trends in team production and inventor allocation, which have changed substantially over time. We then analyze their interaction and aggregate implications for innovation outcomes and welfare by explicitly modeling the role of firm boundaries in idea production.

Using patent data, we document several stylized facts about inventor teams. First, patents developed by teams now account for 80% of all private patents in the U.S., compared with 45% in 1980. Second, around 90% of team collaborations in patenting occur among inventors within the same firm. This underscores the importance of firm boundaries in shaping collaborative idea production. We further find that patenting output per worker, measured by citation-weighted patents, increases with the number of potential collaborators and the intensity of team patenting within the firm. Finally, we document that inventor employment concentration in large firms has risen dramatically over time, and that this rise is more pronounced than the increase in overall employment concentration (Autor et al. 2019) over the same period by a factor of six.

Motivated by these facts, we propose a novel and tractable framework for modeling team-based invention within firms. We build a general equilibrium model of the inventor labor market with heterogeneous firms in the spirit of Hopenhayn (1992) and Melitz (2003), augmented with two new ingredients. First, we explicitly model the process of inventor team formation within firms. Each inventor draws a bilateral productivity for collaborating with every other inventor in the firm and can alternatively work alone with some fixed productivity. The firm optimally assigns inventors across solo and team production to maximize output. As a result, realized inventor productivity is an increasing but concave function of the number of inventor coworkers in the firm, as inventors are increasingly likely to find a productive match with each additional coworker.[1] Second, we incorporate knowledge spillovers across firms that depend on both firm productivity and firm employment size, following Chikis, Kleinman, and Prato (2025). The presence of spillovers implies that the equilibrium firm size distribution need not be efficient.

A key result from our microfoundation for team production is that firms that use team production have higher returns to scale than firms where inventors work alone. Firms' endogenous choice between solo and team production therefore shapes firm size and the allocation of inventors across firms.

Next, we use a combination of direct and indirect inference to estimate how key model parameters, particularly those governing the idea production function for solo inventors and teams, have evolved over time. We then use our model to quantitatively evaluate how these changes affect both the competitive equilibrium allocation and the socially efficient allocation of inventors across firms and assess welfare implications.

We estimate the model using data on innovative firms, inventors, and patenting in the U.S. for two periods: 1976-1999 and 2000-2018. The estimation combines direct production function estimates of the model's idea production function with indirect inference to match the evolution of team patenting, inventor allocation across firms, and firm dynamics. We find that the average productivity of both solo inventors and

[1] We support these modeling assumptions and the implied relationship between inventor productivity and firm size with empirical evidence from the U.S. patent data.

inventor teams has fallen over time, while the returns to scale benefit of teams, which depends on the thickness of the right tail of pairwise inventor productivity draws, has increased.[2] Our model predicts that these changes in the idea production function generate a substantial increase in employment concentration and average firm size, consistent with the data. At the same time, the model predicts higher innovative output but an endogenous decline in knowledge spillovers, consistent with the declining knowledge diffusion hypothesis of Akcigit and Ates (2023).

Finally, we assess the welfare implications of the changes in idea production associated with the rise of patenting teams. We find that inventor misallocation becomes more pronounced as team production expands in the presence of knowledge spillovers. The underlying mechanism is that the measured shifts in the idea production function favor large, highly productive firms, which leads to excessive firm exit and a contraction of smaller firms who generate a disproportionately large amount of knowledge spillovers compared to their innovative output (Chikis, Kleinman, and Prato 2025). These results have implications for the design of R&D policy. In particular, the welfare gains from subsidizing small firms and firm entry are larger in the current environment than in the past.

Related Literature. A growing body of literature examines the match between individual inventors and firms (Akcigit and Goldschlag 2023a; Akcigit and Goldschlag 2023b; Babalievsky 2023; Manera 2022; Liu 2023; Guccione and Roldan-Blanco 2026). A separate strand of literature models firms as collections of coworkers or co-inventors, without incorporating additional dimensions of firm heterogeneity (Herkenhoff et al. 2024; Pearce 2023; Freund 2024). This paper bridges the gap between these two approaches by exploring how inventor team formation varies with firm-level characteristics (in particular idiosyncratic firm productivity, which is intimately connected in

[2] The decline in average solo and team productivity is consistent with recent evidence in Jones (2009), Bloom et al. (2017), and Pearce (2023), which points to increasing difficulty in idea production as a solo inventor and a greater need for combining expertise. The increased benefits of team production are also consistent with rising worker complementarity and assortative matching documented in Freund (2024).

most models, including ours, to employment size) and relates to firm dynamics and inventor (mis)allocation across firms.[3] This connection is important for understanding how changes in the idea production function might have played a role in the recent rise in employment concentration of inventors.

Our model of teamwork within firms contributes to our understanding of the boundary of the firm and optimal scale (Lucas 1978; Chandler 1977). Recent empirical work finds evidence of differences in returns to scale at the firm level within industries (Chan et al. 2025) and evidence that the supply of managers (Engbom et al. 2025), standardization (Argente et al. 2025), and economic development (Bassi et al. 2023; Chen 2023) all influence optimal firm scale. We contribute an additional microfoundation for changing optimal firm scale: idea production in teams, and validate the existence of this explanation using firm level data on inputs (inventors) and output (patents). Our mechanism may extend beyond patenting to other settings in which workers increasingly specialize along the development path. However, patent data provide a uniquely rich environment to observe team collaborations within firms, which is more challenging to measure in other industries.

Our findings also contribute to understanding trends in productivity differences between large and small firms, particularly in the context of research productivity (Andrews, Criscuolo, and Gal 2016; Bessen et al. 2020; Pugsley, Sedlacek, and Sterk 2021; Cavenaile, Celik, and Tian 2025; Olmstead-Rumsey 2025; Akcigit and Ates 2023), and the recent trends of declining business dynamism and missing high-growth young firms (Decker et al. 2014; Decker et al. 2016; Sterk, Sedláček, and Pugsley 2021; Akcigit and Ates 2023).

This paper is also related to the literature on how interactions between individuals shape economic growth (Jones 2009; Lucas and Moll 2014; Akcigit et al. 2018; Prato 2022). Relative to these papers, we abstract from the dynamics of individual-level knowledge accumulation and instead focus on the organizational features that deter-

[3] Jarosch, Oberfield, and Rossi-Hansberg (2021) and Boerma, Tsyvinski, and Zimin (2023) study models of sorting into teams within firms, but not specifically for inventors.

mine how knowledge is produced within firms (Garicano and Rossi-Hansberg 2015). Our model delivers the novel prediction that R&D workers increasingly concentrate in large firms, which highlights the importance of understanding how human capital accumulation for R&D workers operates. If exposure to a diverse set of employers over an inventor's career is a key source of knowledge accumulation, rising concentration in larger firms may pose challenges. On the other hand, the shift from solo invention toward collaboration may accelerate on-the-job human capital accumulation through peer learning within firms (Akcigit et al. 2018). We leave the study of these dynamics for future work.

Finally, several empirical studies highlight the role of inventor teams within firms. Jaravel, Petkova, and Bell (2018) find that the death of a co-inventor negatively impacts subsequent inventor productivity. Bhaskarabhatla et al. (2021) show that a significant share of firm-level heterogeneity in research productivity is driven by differences in inventor quality. Kline et al. (2019) find that firms share around 30% of patenting rents with incumbent high-paid employees. We abstract from these issues and instead focus our attention on how the number of coworkers within a firm affects inventor productivity. We document new facts on patenting output and team size as functions of firm size, and show that organizing inventors into teams raises returns to scale. Our findings also indicate that this effect strengthens over time and carries important welfare implications because of its interaction with the (mis)allocation of workers across firms.

2 Empirical Motivation

2.1 Data Description

Our primary data sources are the United States Patent and Trademark Office (USPTO) PatentsView, Revelio Labs LinkedIn data, and S&P's Compustat. The USPTO PatentsView database tracks the universe of patents granted by the U.S. Patent and Trademark Office (USPTO) from 1976 onward. It provides detailed patent-level information, including application and grant dates, technology class classifications, inventor identifiers, citations, and the names and addresses of patent assignees (firms).

The Revelio Labs LinkedIn data offer worker-level employment histories based on LinkedIn profiles, which is linked to the USPTO inventor data. This allows us to match inventors to firms in years that they do not patent and hence do not appear in the USPTO data.[4]

S&P Compustat provides detailed firm-level data for publicly listed firms, maintained by S&P Global. It includes information such as firm size, industry scope, balance sheets, income and cash flow statements, and stock prices.

We collect utility patents filed between 1976 and 2018, granted by the USPTO. To identify the patents assigned to U.S. public firms and track them by different firm characteristics, we link the USPTO assignee identifier to the Compustat GVKEYs (the firm identifiers in Compustat) using the bridge developed by Ding, Jo, and Kim (2022) and Braguinsky et al. (2023).[5] When a patent is assigned to multiple entities, we only keep the first assignee following the sequence order recorded by USPTO. To measure the quality of patents, we calculate five-year forward citations with vintage fixed effects removed, and obtain the real patent value measures from Kogan et al. (2017) (KPSS values, hereafter) for patents filed by publicly held firms.

2.2 Empirical Findings

Next, we summarize the core set of empirical findings that emerge from our dataset.

Fact 1: Team patents have increased from 50% (1976) to 80% (2018) of patents

First, team-based patenting has become increasingly prevalent over the past five decades. We define a *team patent* as a patent with multiple inventors, and refer to the set of in-

[4]Eventually, we aim to incorporate administrative data from the U.S. Census Bureau to extend our analysis, which is currently in progress.

[5]This bridge is constructed using the standard name and address matching process, complemented by an internet search-aided algorithm following Autor et al. (2020) as well as additional manual matching. The detailed methodology is described in Ding, Jo, and Kim (2022) and Braguinsky et al. (2023).

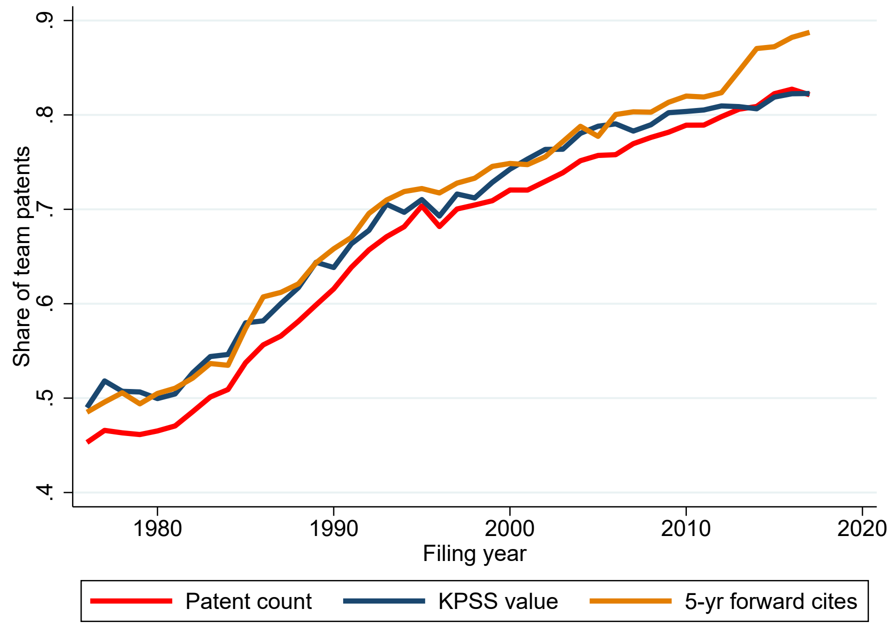


Figure 1: Team Patents Shares

Note: The figure depicts the share of team patents at Compustat firms from 1976 to 2018. A team patent is defined as one assigned by multiple inventors. The number of patents is counted unweighted (red line), weighted by KPSS values from Kogan et al. (2017) (navy line), and weighted by five-year forward citations with vintage fixed effects removed (orange line).

ventors listed on the patent as a *team*.

In our data, the share of team patents has risen from below 50% in 1976 to above 80% in 2018. This upward trend holds either measured in patent counts, market-based patent value (KPSS values), or five-year forward citations. These patterns are illustrated in Figure 1.[6]

Fact 2: 90% of team patents are by inventors at the same firm

Second, we find that the vast majority of team collaboration in innovation occurs within firm boundaries rather than across firms. This highlights the role of firms as the critical platform for team production.

[6]In this figure, we restrict our sample to Compustat firms with patent filings given the availability of KPSS values and to maintain consistency of the sample across series. The results are robust when using the full set of USPTO assignees.

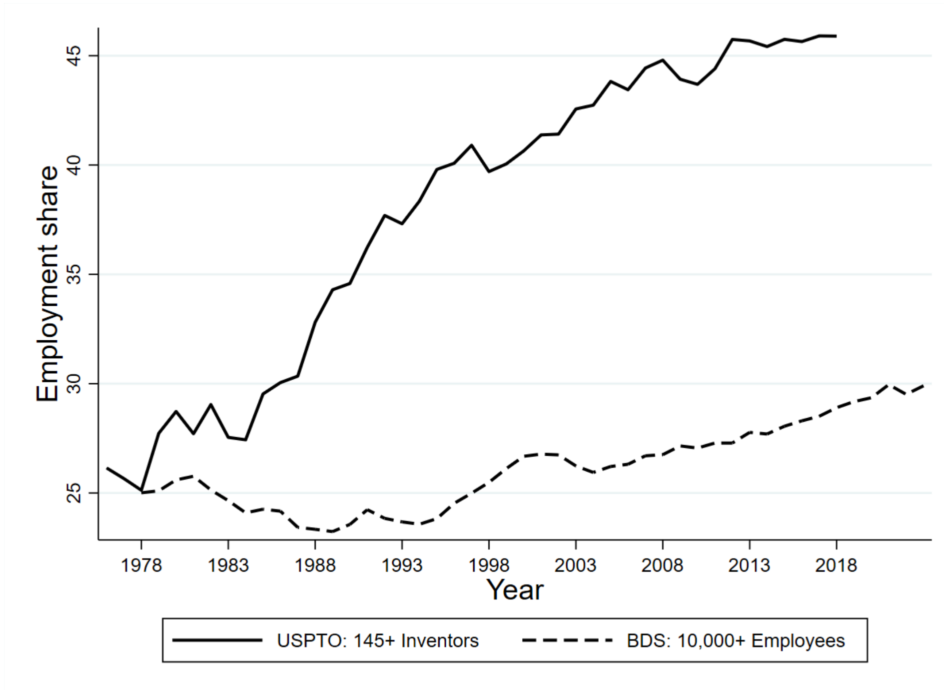


Figure 2: Employment Concentration: Inventors and Overall Employment

Note: The figure plots the employment share of inventors at the largest firms (ranked by the number of inventors), indexed to match the employment share of overall employment at the largest firms (ranked by total employment) in 1978. Inventor employment is measured using patent data in USPTO to count inventors at the firm–year level, while overall employment shares are constructed from the Census Bureau's Business Dynamics Statistics (BDS).

In our data, we identify an inventor's affiliation in a given year as the assignee (firm) with which the inventor files the largest number of patents in that year. In the event of a tie, we assign the affiliation to the firm associated with the patent having the latest filing date. Using this definition, 89.5% of team patents in our sample are produced by inventors affiliated with the same firm. This indicates that most team-based innovation reflects within-firm collaboration, and represents a lower bound given the way we define inventor affiliation to at most one firm in a given year.

Fact 3: Inventor employment concentration has increased much more than overall employment concentration

A growing literature analyzes the causes and consequences of rising employment concentration in the United States (Autor et al. 2017; Rossi-Hansberg, Sarte, and Trachter 2020). Akcigit and Ates (2023) were the first to point out the increase in employment concentration specifically among inventors and Akcigit and Goldschlag (2023b) provide additional supporting evidence. To our knowledge, we are the first to provide a direct comparison between overall employment concentration and concentration in inventor employment.

To do this, we use the USPTO data to match inventors to firms (assignees in the patent data).[7] A standard measure of overall employment concentration is the share of total employment accounted for by firms with more than 10,000 employees reported by the Census Business Dynamics Statistics (Autor et al. 2017). We set a similar threshold for inventor employment by matching the share of inventor employment at large innovative firms in 1978 to the share of overall employment at firms with more than 10,000 employees in 1978. This procedure gives us a threshold of 145 inventors for defining large innovative firms.

Holding fixed this threshold over time, the inventor employment share of large innovative firms has risen about six times more than the corresponding overall employment share of large firms. Figure 2 presents this pattern. Our model in the following section explores how this divergence can arise from changes in optimal innovative firm size driven by changes in the idea production function that favor knowledge production in teams.

[7] This is the same approach taken by Akcigit and Goldschlag (2023b) and means that our measure of inventors includes only those who patent in a given year. The Revelio Labs data provides a broader coverage of inventors, including those who do not patent, and we use it as a supplementary source. However, this data does not span a sufficiently long time horizon to study long-run trends in concentration.

3 Theoretical Framework

To explain the trends documented in Section 2 and assess their implications for the allocation of inventors across firms, we develop a theoretical framework to study the problem of heterogeneous innovative firms hiring inventors in general equilibrium.

Our model has the following three key ingredients: first, we introduce firm heterogeneity in productivity to generate dispersion in firm size as is standard in the firm dynamics literature (Hopenhayn 1992; Melitz 2003); second, we provide a novel microfoundation for the firm's internal allocation of inventors between solo and team-based idea production and endogenize the fraction of team patenting; and third, we allow for productivity spillovers across firms that depend both on a firm's intrinsic productivity and its employment size following Chikis, Kleinman, and Prato (2025).

We begin by specifying inventor-level productivity, which depends on whether the inventor works alone or in a team. We then describe the firm's problem of hiring inventors and assigning them across solo and team production. Finally, we define and characterize the equilibrium. Appendix A.1 provides additional empirical evidence that supports the key modeling assumptions and results.

3.1 Inventor Productivity

Inventor i has the following productivity:

$$\begin{cases} z_{ii} = \bar{z} & \text{if working alone} \\ z_{ij} \sim P(x_{min}^{team}, \gamma^{team}) & \text{if working with another inventor } j, \end{cases} \tag{3.1}$$

where an inventor can either work alone ("solo") or collaborate with another inventor within a firm ("team"). When working alone, inventor productivity is constant at $\bar{z}$. Under team production, each inventor can form a potential collaboration with every other worker in the firm by drawing a symmetric, pairwise productivity z_{ij} from a Pareto distribution with scale and shape parameters $(x_{min}^{team}, \gamma^{team})$.[8] These draws reflect the potential gains from collaboration: inventors may combine their expertise in highly productive ways, for instance through complementarities in skills or knowledge. The stochastic structure of pairwise productivity implies heterogeneity in match quality within the firm. Thus, not all inventors pair equally well, and not all collaborations are equally productive in the firm. Our assumptions imply that inventors are ex ante homogeneous.

Suppose the firm's objective is to maximize the aggregate productivity of its inventors (which corresponds to profit maximization in the following section). The firm's problem can be broken down into two steps. First, for each inventor, the firm determines the most productive team collaboration among all feasible pairings within the firm. Second, it compares this maximal team productivity with the productivity of working alone, $\bar{z}$, and assigns the inventor to the mode that yields higher productivity.

We assume that the firm can observe the productivity of all potential collaborations among their workers and can assign each inventor to work with the coworker who yields the highest pairwise productivity. Let z_i^{team} denote the maximal team productivity available to inventor i within the firm. In a firm with n inventors, the realized team productivity for inventor i is given by

$$z_i^{team} = \max\{z_{i1}, z_{i2}, ..., z_{in}\}. \tag{3.2}$$

Given the Pareto distribution of pairwise productivity, the expected team productivity of an inventor in a firm of size n is

$$\mathbb{E}(z_i^{team}) = x_{min}^{team}\Gamma\left(1 - \frac{1}{\gamma^{team}}\right) n^{\frac{1}{\gamma^{team}}}. \tag{3.3}$$

See Appendix A.2.1 for the derivation. Note that this property also holds with a contin-

[8]For simplicity, we assume that each potential collaboration consists of only two workers and abstract from serial collaboration or team-specific human capital accumulation. These assumptions are consistent with the empirical evidence provided in Appendix A.1.

uum of workers by interpreting n as the mass of inventors employed by the firm rather than a discrete headcount. See Appendix A.2.2 for details.

Equation (3.3) implies that expected team productivity is increasing in firm size n, the scale parameter x_{min}^{team}, and in the thickness of the upper tail (i.e., the inverse of the tail parameter γ^{team}) of the Pareto distribution of pairwise productivity draws. Figure 3 illustrates these features.

At the worker level, team productivity exceeds solo productivity when the worker belongs to a sufficiently large firm (with size n). In particular, there exists a firm-size threshold such that inventors in firms with size above the cutoff achieve higher expected productivity in teams than under solo production. Figure 4 depicts this relationship and the cutoff. Consistent with this, Appendix Table A1 documents a positive, economically and statistically significant relationship between firm size (measured by the number of inventors) and the share of team patents in the firm's total patent count.

3.2 Firm Idiosyncratic Productivity

Given this structure, we can characterize firm-level productivity. Let a firm's productivity in producing ideas, z_f, be determined by the average productivity of its inventors, augmented by an idiosyncratic firm-level shock ε_f. We assume that ε_f is drawn from a Pareto distribution g with scale and shape parameters (x_{min}^f, γ^f).

Since workers are ex-ante homogeneous, the average worker productivity within the firm coincides with the productivity of any individual worker employed by the firm. Firm productivity therefore inherits the endogenous team-versus-solo allocation at the worker level, scaled by the firm-specific productivity. Equation (3.4) characterizes realized firm productivity.

$$z_f = \begin{cases} \bar{z}\varepsilon_f & \text{w/ solo production} \\ \underbrace{x_{min}^{team}\Gamma(1 - \frac{1}{\gamma^{team}})n_f^{\frac{1}{\gamma^{team}}}}_{=\mathbb{E}(z_i^{team})}\varepsilon_f & \text{w/ team production.} \end{cases} \tag{3.4}$$

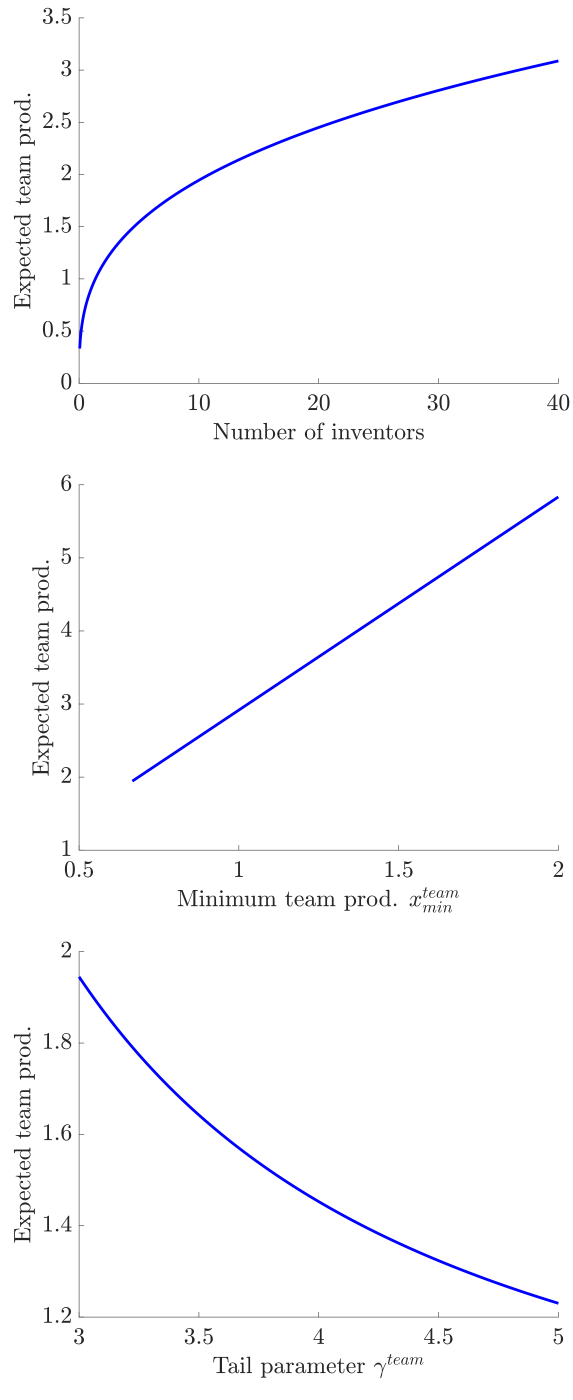


Figure 3: Expected Team Productivity $\mathbb{E}(z_{it}^{team})$

Note: The figure illustrates the expected team productivity by firm size (top) and by different values of scale (middle) and tail (bottom) parameters from the Pareto distribution.

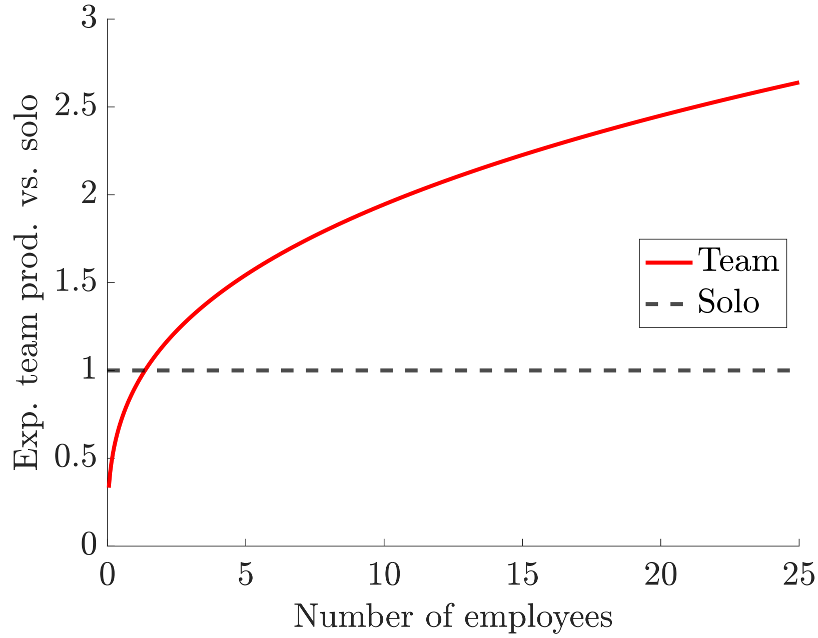


Figure 4: Team vs Solo Productivity

Note: The figure compares worker-level productivity under team and solo production. The red solid line indicates team production, while the black dashed line shows solo production.

Relative to a standard framework with exogenous firm-level productivity, our microfoundation implies that firm productivity is endogenous to the choice of solo or team production and, importantly, to the number of inventors the firm hires conditional on choosing team production. As a result, firm size directly affects productivity through endogenous team formation.

3.3 Knowledge Spillovers

There are knowledge spillovers across firms in the economy. This is motivated by the finding that the social returns to innovation are substantially larger than private returns (Bloom, Schankerman, and Van Reenen 2013). We formalize spillovers in the model in the same way as Chikis, Kleinman, and Prato (2025). In particular, spillovers come from a mix of firm productivities z_f and size n_f as follows:

$$\bar{z}^{spillover} = \left(\int_{f \in F} z_f^{1-\beta} n_f^{\beta} df \right)^{\theta}, \tag{3.5}$$

where F is the measure of active firms. $\theta > 0$ governs the importance of knowledge spillovers for firm output, and $\beta \in [0,1]$ governs how much firm size (n_f) matters compared to firm productivity (z_f) for generating spillovers. In the extreme cases, if $\beta = 1$, only the total number of inventors matters for spillovers, not their allocation to particular firms. If $\beta = 0$, the allocation of workers also does not matter because all that matters is the average productivity of firms regardless of their size. For intermediate cases, β governs the concavity of the spillover in labor, with more concavity (lower β) implying that the marginal social returns to additional inventors in a given firm are low.[9]

3.4 Firm Problem

Given their idiosyncratic productivity draw, innovative firms solve a static profit maximization problem. Producing requires a fixed operating cost c_f. Firms simultaneously choose whether to produce or exit, and conditional on operating, choose the number of inventors to hire and whether to organize these inventors into solo or team production. The following characterizes the firm's profit function:

$$\Pi(\varepsilon_f) = \max \left\{ 0, \Pi^{solo}(\varepsilon_f), \Pi^{team}(\varepsilon_f) \right\}, \tag{3.6}$$

where

$$\Pi^{solo}(\varepsilon_f) = \max_{n_f^{solo}} \bar{z}^{spillover} \bar{z} \varepsilon_f (n_f^{solo})^{\alpha} - w n_f^{solo} - c_f, \tag{3.7}$$

$$\Pi^{team}(\varepsilon_f) = \max_{n_f^{team}} \bar{z}^{spillover} x_{min}^{team} \Gamma(1 - \frac{1}{\gamma^{team}}) \varepsilon_f (n_f^{team})^{\frac{1}{\gamma^{team}} + \alpha} - w n_f^{team} - c_f. \tag{3.8}$$

[9] Chikis, Kleinman, and Prato (2025) use data on citations patterns to the same firm in different locations where the firm has plants of varying sizes and estimate $\beta = 0.25$.

Throughout, we normalize the "price" of innovative output to 1. The model could be extended to include a production sector where ideas generated by the firm's R&D lab are used to produce physical output.

Profit maximization creates a cutoff for ε_f above which firms choose team production. Let $\tilde{\varepsilon}^{team}$ denote this cutoff. The fixed cost creates another cutoff for firm productivity, $\tilde{\varepsilon}^{exit}$, below which firms endogenously exit. Let $d = \{0, 1\}$ denote the exit choice, which equals to 1 if firm exits.

The optimal inventor employment size of firms under solo and team production can be derived as follows:

$$n^*(\varepsilon_f) = \begin{cases} \left(\frac{\alpha \bar{z}^{spillover} \bar{z} \varepsilon_f}{w}\right)^{\frac{1}{1-\alpha}} & \text{w/ solo (if } \tilde{\varepsilon}^{exit} < \varepsilon_f < \tilde{\varepsilon}^{team}) \\ \left(\frac{(\frac{1}{\gamma^{team}}+\alpha)\bar{z}^{spillover} x_{min}^{team} \Gamma(1-\frac{1}{\gamma^{team}})\varepsilon_f}{w}\right)^{\frac{1}{1-(\frac{1}{\gamma^{team}}+\alpha)}} & \text{w/ team (if } \tilde{\varepsilon}^{team} < \varepsilon_f), \end{cases} \tag{3.9}$$

where $\tilde{\varepsilon}^{exit}$ and $\tilde{\varepsilon}^{team}$ are determined by the following equations, respectively,

$$\begin{aligned} \tilde{\varepsilon}^{exit} &= \frac{c_f^{1-\alpha} w^{\alpha}}{(1-\alpha)^{(1-\alpha)} \alpha^{\alpha} \bar{z}^{spillover} \bar{z}}, \\ \tilde{\varepsilon}^{team} &= \left(\frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} \bar{z}^{\frac{1}{1-\alpha}}}{(1-(\frac{1}{\gamma^{team}}+\alpha))(\frac{1}{\gamma^{team}}+\alpha)^{\frac{(\frac{1}{\gamma^{team}}+\alpha)}{1-(\frac{1}{\gamma^{team}}+\alpha)}} (x_{min}^{team}\Gamma(1-\frac{1}{\gamma^{team}}))^{\frac{1}{1-(\frac{1}{\gamma^{team}}+\alpha)}}} \right)^{\gamma^{team}(1-\frac{1}{\gamma^{team}}-\alpha)(1-\alpha)} \\ &\times \frac{w}{\bar{z}^{spillover}}. \end{aligned}$$

3.5 Firm Entry

A large mass of potential entrants can enter by paying an entry cost c_e before drawing their productivity. The following free entry condition must hold in equilibrium:

$$\int \Pi(\varepsilon_f) dg(\varepsilon_f) - c_e = 0, \tag{3.10}$$

which pins down the mass of firms M. Entrants draw their idiosyncratic productivities from the same Pareto distribution g of incumbent productivities.

3.6 Labor Market Clearing

There is a fixed supply of inventors with measure 1. Labor market clearing requires that the aggregate inventor labor demand is equal to the supply:

$$M\Big(\int_{\tilde{\varepsilon}^{team}} n^{team}(\varepsilon_f;w)dg(\varepsilon_f) + \int_{\tilde{\varepsilon}^{exit}}^{\tilde{\varepsilon}^{team}} n^{solo}(\varepsilon_f;w)dg(\varepsilon_f)\Big) = 1. \tag{3.11}$$

This pins down the equilibrium wage w in the economy.

3.7 Equilibrium

An equilibrium is a collection of $\Big\{\{n_f^{solo}, n_f^{team}, d_f, \Pi_f\}_f, \tilde{\varepsilon}^{exit}, \tilde{\varepsilon}^{team}, w, M\Big\}$, where firms solve (3.6)-(3.9); the free-entry condition (3.10) holds; and the labor market clearing condition (3.11) is satisfied.

4 Social Planner's Problem

We next consider how the efficient allocation of inventors across firms differs from the allocation in competitive equilibrium. In this section, for expositional simplicity, we focus on a simple case where firms operate exclusively under either solo or team production and abstract from firm entry and exit by assuming a fixed measure one of firms. This setting provides intuition for the differences between the efficient allocation and the competitive equilibrium under solo and team production and how the magnitude of misallocation varies across the two production schemes. In the quantitative model (Section 6.3), we solve the full planner's problem, which jointly determines firm size n_f, the choice between solo and team production, the mass of firms M, and the firm exit cutoff (equivalently, the mass of active firms).

We first characterize the wedges between social and private marginal product of inventors across firms with different productivity levels in each type of production (solo or team). We then compare the magnitudes of the wedges across solo and team production.

The planner allocates inventors across firms to maximize total output, internalizing spillovers and the resource constraint. The planner's problem can be written as follows:

$$\max_{n_f} Y = \int_0^1 \bar{z}^{spillover} z_f n_f^{\alpha} df, \tag{4.1}$$

subject to (3.4), (3.5), and

$$\int_0^1 n_f df = \bar{N}. \tag{4.2}$$

We define a firm-level wedge τ_f as the gap between the social marginal product of inventor labor at firm f (MP_f^{SP}) and the private marginal revenue product (MP_f^{CE}) perceived by the firm:

$$MP_f^{SP} = (1+\tau_f) MP_f^{CE}. \tag{4.3}$$

4.1 Solo Production

We first solve the for the optimal allocation of inventors in the case where all firms operate under solo production.

Under solo production, the planner faces the following marginal product of labor:

$$MP_f^{solo,SP} = \theta\beta(\bar{z}^{spillover})^{-\frac{1}{\theta}}(\bar{z}\varepsilon_f)^{1-\beta} n_f^{\beta-1} Y + \alpha \bar{z}^{spillover}(\bar{z}\varepsilon_f) n_f^{\alpha-1}, \tag{4.4}$$

and the counterpart in competitive equilibrium with all solo production is

$$MP_f^{solo,CE} = \alpha \bar{z}^{spillover} \bar{z}\varepsilon_f n_f^{\alpha-1}. \tag{4.5}$$

Using (4.3), (4.4) and (4.5), the wedge under solo production is:

$$\tau_f^{solo} = \frac{\theta\beta Y}{\alpha(\bar{z}^{spillover})^{1+\frac{1}{\theta}}} (\bar{z}\varepsilon_f)^{-\beta} n_f^{\beta-\alpha}. \tag{4.6}$$

The wedge includes a common component for all firms and a firm-specific component that depends on idiosyncratic productivity ε_f. In the spirit of Hsieh and Klenow (2009), we evaluate this wedge at the competitive equilibrium firm size distribution and obtain the following result.

Proposition 1. *If $\alpha, \beta \in (0,1)$, firm-level wedges are decreasing in firm productivity ε_f and size n_f under solo production.*

Proof. See Appendix A.3.1. □

Proposition 1 implies misallocation of inventor labor across firms. All firms are inefficiently small because of knowledge spillovers, but the planner values additional inventors at small firms the most.

4.2 Team Production

Under team production, the planner faces the following marginal product of labor:

$$MP_f^{team,SP} = \theta\big(\beta + \frac{1}{\gamma^{team}}(1-\beta)\big)(\bar{z}^{spillover})^{-\frac{1}{\theta}}\big(x_{min}^{team}\Gamma(1-\frac{1}{\gamma^{team}})\varepsilon_f\big)^{1-\beta} n_f^{(\beta-1)(1-\frac{1}{\gamma^{team}})} Y$$
$$+ x_{min}^{team}\Gamma(1-\frac{1}{\gamma^{team}})\big(\alpha + \frac{1}{\gamma^{team}}\big)\bar{z}^{spillover}\varepsilon_f n_f^{\alpha+\frac{1}{\gamma^{team}}-1}, \quad (4.7)$$

and the counterpart in competitive equilibrium is

$$MP_f^{team,CE} = x_{min}^{team}\Gamma(1-\frac{1}{\gamma^{team}})\big(\alpha + \frac{1}{\gamma^{team}}\big)\bar{z}^{spillover}\varepsilon_f n_f^{\alpha+\frac{1}{\gamma^{team}}-1}. \quad (4.8)$$

Using (4.3), (4.7) and (4.8), we get the general expression for the wedge under team production as follows:

$$\tau_f^{team} = \frac{\theta\big(\beta + (1-\beta)\frac{1}{\gamma^{team}}\big)Y}{\big(\alpha + \frac{1}{\gamma^{team}}\big)\big(\bar{z}^{spillover}\big)^{1+\frac{1}{\theta}}}\big(x_{min}^{team}\Gamma(1-\frac{1}{\gamma^{team}})\varepsilon_f\big)^{-\beta} n_f^{\beta(1-\frac{1}{\gamma^{team}})-\alpha}. \quad (4.9)$$

The following proposition shows the evaluation of the wedges at the competitive equilibrium firm size distribution under team production.

Proposition 2. *If* $\alpha + \frac{1}{\gamma^{team}} \in (0,1)$ *and* $\beta \in (0,1)$*, firm-level wedges are decreasing in firm productivity* ε_f *and size* n_f *under team production.*

Proof. See Appendix A.3.2. □

Proposition 2 establishes that team production also generates misallocation of inventor labor across firms. The following proposition compares the magnitude of the wedges under solo production and team production, highlighting how team formation increases the wedges between social and private returns.

Proposition 3. *If* $\alpha + \frac{1}{\gamma^{team}} \in (0,1)$*,* $\beta \in (0,1)$*, and* $\frac{\beta}{1-\beta} < \alpha$*, firms have smaller wedges under solo production than under team production.*

Proof. See Appendix A.3.3. □

5 Calibration

We estimate two sets of model parameters, one for the early period (pre-2000) and the other for late period (post-2000) of our sample. A subset of parameters is directly estimated and disciplined by using firm-level data on patent output, inventor employment, the within-firm share of team-based patenting compared to solo patenting, the patenting entry rate, and average firm size (in terms of number of inventors). The remaining parameters are externally determined, either through normalization or taken from the literature.

5.1 Estimating the Idea Production Function

First, to pin down α, the baseline returns to scale in inventor labor under solo production, γ^{team}, the additional effect of team-based production on returns to scale, x^{team}_{min}, the minimum of the team productivity distribution, and the solo productivity parameter $\bar{z}$, we estimate the model's firm-level idea production function in the data.

Specifically, we implement the following triple-difference regression using the universe of patenting firms in the USPTO data over the period 1976–2018:

$$\log(pats_{ft}) = \mu_f + \alpha \log(n_{ft}) + \frac{1}{\gamma^{pre}}(\log n_{ft} \times \mathbb{I}(team_{ft})) + \frac{1}{\gamma^{post}}(\log n_{ft} \times \mathbb{I}(team_{ft}) \times Post_t) + \nu_{ft}. \tag{5.1}$$

Our baseline measure of firm output is patent counts, where $pats_{ft}$ denotes the number of patents filed by firm f in year t. The term μ_f captures a firm fixed effect, corresponding to $\log(\varepsilon_f)$ in the model, and n_{ft} is the number of (patenting) inventors employed by firm f in year t. The indicator $\mathbb{I}(team_{ft})$ equals one if the firm's share of team patents $\left(\frac{\text{team patents}_{ft}}{\text{total patents}_{ft}}\right)$ exceeds the sample median (0.88) in year t, and zero otherwise. $Post_t$ is a dummy equal to one for the period 2000-2018. All base effects and interactions of these regressors are included in the regression in addition to the terms in equation (5.1).[10]

The results are reported in Column 1 in Table 1 and the resulting structural parameter estimates are summarized in Table 2. We find evidence of decreasing returns to scale in inventor labor in the baseline period (pre-2000), with an estimate of $\alpha = 0.757 < 1$ and no evidence of a change in returns to scale for solo firms over time (the coefficient on $\log(n_{ft}) \times Post_t$ is close to zero and not statistically significant).

Consistent with the model, team production increases returns to scale with the estimate of $\frac{1}{\gamma^{pre}} = 0.0394$. This implies $\gamma^{team} = 25.381$ in the early period. Together the estimates of α and γ^{team} imply that team based firms still have decreasing returns to scale, but less-decreasing than solo-based firms where inventors tend to work alone (around 0.8 instead of 0.76 for the early period).

The positive coefficient on the triple interaction indicates that $\frac{1}{\gamma^{team}}$ rises in the later period (post-2000), implying a thicker right tail of pairwise productivity draws and

[10] Identification relies on variation in the choice of using solo and team production, conditional on firm fixed effects μ_f and inventor employment size n_{ft}. A potential concern is that this choice may be correlated with unobserved determinants of output. We currently exploring the use of various instruments for this choice including the past teamwork experience of new inventors at the firm and communication costs within the firm (Bloom et al. 2014; Jiang 2024).

a greater upside potential of using teams, possibly due to rising complementarities.[11] Quantitatively, the late-period estimate is $\gamma^{team} = \frac{1}{0.0394+0.0312} = 14.164$.

Finally, the negative coefficients associated with the dummies $Post_t$ and $Post_t \times \mathbb{I}(team_{ft})$ indicate a general decline in patenting productivity across firms in the later part of the sample period. This pattern may reflect a broader decline in research productivity, in line with the evidence that ideas have become harder to find (Bloom et al. 2017). We normalize solo-based firms in the early period (as the base group in the regression) and set $\bar{z} = 1 - 0.019 = 0.981$ for the late period. The early period value of minimum team productivity x_{min}^{team} is internally calibrated to match the team patent share in the early period, and then we use the coefficient on $Post_t \times \mathbb{I}(team_{ft})$ to infer the decline in minimum team productivity between the two periods. The details are in Appendix C.1.

Robustness. Table 1, Columns 2 and 3, report estimates from the same specification using restricted subsamples of firms. Column 2 focuses on firm-year observations with at least two inventors at the firm in that year. Column 3 limits the sample to public firms, which allows us to control for capital inputs into R&D activities using the Peters and Taylor (2017) measures of intangible capital (capitalized selling and general administrative and R&D expenses). Controlling for these inputs addresses concerns that omitted inputs (capital) that are positively correlated with inventor employment or team usage could bias the estimated returns to scale upward. Across both subsamples, our main findings remain qualitatively robust.

In Appendix C.2, Table C4, we re-estimate Equation 5.1 using forward citations instead of patent counts as the measure of firm output, adjusted for technology-class fixed effects for citations. In this case, the returns-to-scale advantage of team production is statistically significant only in the post-2000 period but the general patterns remain similar to the baseline results.

[11] Consistent with the idea of rising complementarities across inventors over time, we document in Appendix C.3 that patenting teams increasingly combine inventors from different technological subfields, defined by the technology class in which an inventor patents most frequently.

Table 1: Returns to Scale in Team Production

	(1) All	(2) $n > 1$	(3) Public Firms
$\log(n_{ft})$	0.757***	0.780***	0.818***
	(0.002)	(0.003)	(0.004)
$\log(n_{ft}) \times \text{Team}_{ft}$	0.039***	0.038***	0.0875***
	(0.004)	(0.004)	(0.006)
$\log(n_{ft}) \times \text{Team}_{ft} \times \text{Post}_t$	0.031***	0.001	0.011*
	(0.004)	(0.005)	(0.006)
Team_{ft}	-0.677***	-0.647***	-0.732***
	(0.004)	(0.006)	(0.008)
Post_t	-0.019***	-0.108***	-0.058***
	(0.003)	(0.007)	(0.009)
$\log(n_{ft}) \times \text{Post}_t$	0.003	0.029***	0.019***
	(0.002)	(0.003)	(0.004)
$\text{Team}_{ft} \times \text{Post}_t$	-0.049***	0.041***	-0.015
	(0.005)	(0.007)	(0.011)
$\log(K_{ft}^{intan.})$			-0.012***
			(0.002)
Constant	0.065***	-0.000	0.081***
	(0.002)	(0.006)	(0.014)
R-squared	0.704	0.720	0.817
Obs.	1105437	759085	131166
Firm FE	Yes	Yes	Yes

Notes: Source: USPTO, 1976-2018. Column (1) includes all firm–year observations. Column (2) restricts the sample to firm–year observations with at least two inventors. Column (3) includes public firms with available intangible capital data from Peters and Taylor (2017). See equation (5.1) for details. Standard errors are reported in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Another concern is that inventor employment is measured with error, since the USPTO data capture only inventors who successfully patent in a given year. To address this issue, we use Revelio Labs data on inventor employment histories to construct a better measure of inventors per firm. However, these data are reliable only from around 2008 onward. Appendix Table C5 reports the corresponding estimates. While the baseline returns-to-scale parameter (α) is lower in this sample, the estimated team effects (γ^{team}) in the pre- and post-periods (2008–2014 versus 2014–2018) remain qualitatively consistent with our main findings.

Table 2: Calibration

Panel A: Early Period (1976-1999)			
Parameter	Definition	Value	Target/Source
α	RTS, solo firms	0.757	Table 1
γ^{team}	Higher RTS, team firms	25.381	Table 1
$\bar{z}$	Solo prod.	1	Normalization
x^{team}_{min}	Location for team prod.	0.887	Share of team patents
x^{f}_{min}	Location for ε	1	Normalization
γ^{f}	Tail for ε	6.101	Emp. share of large firms
β	scale for n in spillovers	0.250	Chikis, Kleinman, and Prato (2025)
θ	overall spillovers	0.080	Chikis, Kleinman, and Prato (2025)
c_e	entry cost	0.876	Patenting entry rate
c_f	fixed cost	0.309	Average firm size
Panel B: Late Period (2000-2018)			
Parameter	Definition	Value	Target
α	RTS, solo firms	0.757	Table 1
γ^{team}	Higher RTS, team firms	14.164	Table 1
$\bar{z}$	Solo prod.	0.981	Table 1
x^{team}_{min}	Location for team prod.	0.827	Table 1
x^{f}_{min}	Location for ε	1	Normalization
γ^{f}	Tail for ε	6.101	Fixed to early period
β	scale for n in spillovers	0.250	Chikis, Kleinman, and Prato (2025)
θ	overall spillovers	0.080	Chikis, Kleinman, and Prato (2025)
c_e	entry cost	0.876	Fixed to early period
c_f	fixed cost	0.309	Fixed to early period

Notes: This table reports the parameter values for the calibrated model in the early (Panel A) and late period (Panel B) and the target or source for each parameter value.

5.2 Other Parameters: Early Period

Table 2, Panel A reports the calibrated parameters for the early period. We normalize the productivity of solo inventors, $\bar{z}$, and the lower bound of the firm-level productivity distribution, x^{f}_{min}, to one. The values for the knowledge spillover function are from Chikis, Kleinman, and Prato (2025), who estimate β in several different ways using firm's spatial variation in size, arriving at values between $[0.2, 0.3]$. They also use a variety of approaches to estimate θ, arriving at values between $[0.04, 0.12]$, which is also consistent with other estimates of this parameter for different samples and time periods (Greenstone, Hornbeck, and Moretti 2010; Moretti 2021).

We internally calibrate four parameters, $(x_{\min}^{team}, \gamma_f, c_e, c_f)$, to match four empirical moments. The fit for these targets is reported in Table 3. First, we target the share of team patents over 1976–1999, equal to 57.8 percent (see Figure 1), which pins down $x_{\min}^{team}$. Second, the Pareto tail parameter of the firm productivity distribution is chosen to match the employment share of large firms (with 145 inventors or more), which was 32.8% in the early period (Figure 2). Third, we target the entry rate of patenting firms, defined as the share of entrants (age 0) among all patenting firms using Business Dynamics Statistics (BDS) data over 1978–1999, which yields an average annual entry rate of 2.7 percent.[12] This moment pins down c_e. Finally, we choose c_f match the average inventor employment size of patenting firms in the USPTO data over 1976–1999, equal to 7.1 inventors.

5.3 Other Parameters: Late Period

Our central question is whether changes in the idea production function can account for the observed rise in inventor employment concentration. We hold fixed the parameters related to industry primitives (the productivity distribution parameter γ_f, the entry cost c_e and the fixed cost c_f) and vary the parameters related to the idea production function using the directly estimated values from equation (5.1). Panel B of Table 2 reports the parameter values for the late period.

6 Quantitative Results

6.1 Model Fit and Aggregate Implications

Table 3 shows that the calibrated model closely matches the targeted moments for the early period. Table 4 reports the model predicted levels of and changes in various

[12]The BDS data are available starting in 1978.

Table 3: Targeted Moments For the Early Period (1976-1999)

Target Moment	Data	Model
Share of team patents	57.8	57.8
Employment share of firms with $n > 145$	32.8	32.8
Patenting entry rate (annual)	2.6	2.6
Average firm size (inventors)	7.1	7.1

Notes: This table reports the targeted moments in the data (in the first column) and their corresponding values implied by the model (in the second column). The data sources are the USPTO (1976-1999) and the U.S. Census Bureau Business Dynamics Statistics (BDS, 1978-1999).

Table 4: Model Predicted Changes in Aggregate Moments

	Early	Late	Change
Inventor wage w	0.603	0.705	16.9
Prod. cutoff (exit)	1.004	1.257	25.1
Prod. cutoff (team)	1.579	1.710	8.3
Share of solo-based firms	93.7	84.7	-9.0
Share of team patents	57.8	91.7	33.9
Employment share of firms with $n > 145$	32.8	79.2	46.4
Average firm size	7.1	25.1	253.7
Spillovers $z^{spillover}$	0.889	0.815	-8.3
Output (normalized)	1.000	1.110	11.0

Notes: This table reports the model-implied changes in aggregate moments between the early and late periods. The first two columns are the moments for the early and late periods, respectively, and the third column reports the corresponding percent or percentage point change relative to the early-period benchmark.

aggregate moments over time due to the estimated changes in the idea production function, including various non-targeted moments.

The model predicts a 17% increase in inventor wages, consistent with the evidence in Ekerdt and Wu (2025), who document a rise of roughly 15 percentage points in the wage premium for researchers since 1970. The higher wage increases both the equilibrium exit cutoff and the cutoff for using teams.

The model implies a decline in the share of firms operating with solo production of 9 percentage points. Consistent with this prediction, the data show a decline of approximately 12 percentage points in the share of innovative firms with only one inventor (from 39% to 27% of firm–year observations). Using our empirical definition of solo-based firms from the regression (those with below-median team patent share), the share of solo-based firms declines by 15 percentage points, from 60% to 45%. Average firm size increases from 7.1 to 25.1, whereas in the data average size rose to 10.8 inventors per firm. The model also predicts a substantial rise in employment concentration due to changes in the idea production function. The employment share of large firms (defined as those with 145 or more inventors) increases from 32.8% to 79.1%, compared with an increase to 44.1% in the data.[13]

Together, these changes reduce knowledge spillovers by 8.3%. This suggests a possible microfoundation for the declining knowledge diffusion hypothesis of Akcigit and Ates (2023): shifts in the idea production function generate greater misallocation of inventors across firms by concentrating inventors in large firms, which in turn dampens spillovers since the returns to additional inventors in large firms in terms of additional knowledge spillovers are low. However, the rise in returns to scale generated by team production increases output despite the decline in spillovers across firms. We next decompose the role of individual idea production parameters in more detail and return to the issue of declining knowledge spillovers and misallocation in Section 6.3.

6.2 Decomposition of Aggregate Outcomes

This section decomposes the contribution of each estimated change in the idea production function parameters individually $(\bar{z}, x_{min}^{team}, \gamma^{team})$ to the evolution of key aggregate moments: the share of team patents, employment concentration, as well as innovative output and knowledge spillovers. This exercise isolates the role of each parameter shift and clarifies the mechanisms through which they affect the aggregate outcomes. Table 5 shows the results. Below, we discuss the effects of the parameter changes for each aggregate variable in turn.

[13] Appendix B.1 reports two additional experiments where the exogenous firm productivity distribution is allowed to vary, either jointly with the idea production parameters, or on its own, to match the rise in employment concentration.

Table 5: Decomposing the Role of Idea Production Function Changes

	Early	Late	$\bar{z}$ only	Change
Inventor wage w	0.603	0.705	0.593	-1.8
Prod. cutoff (exit)	1.004	1.257	1.017	1.3
Prod. cutoff (team)	1.579	1.710	1.414	-10.4
Share of solo-based firms	93.7	84.7	86.6	-7.0
Share of team patents	57.8	91.7	68.3	10.5
Employment share of firms with $n > 145$	32.8	79.2	34.3	1.5
Average firm size	7.1	25.1	7.8	10.5
Spillovers $z^{spillover}$	0.889	0.815	0.883	-0.7
Output (normalized)	1.000	1.110	0.977	-2.3
	Early	Late	x_{min}^{team} only	Change
Inventor wage w	0.603	0.705	0.593	-3.2
Prod. cutoff (exit)	1.004	1.257	1.000	1.3
Prod. cutoff (team)	1.579	1.710	2.341	-10.4
Share of solo-based firms	93.7	84.7	99.4	5.8
Share of team patents	57.8	91.7	31.1	-26.7
Employment share of firms with $n > 145$	32.8	79.2	27.6	-5.3
Average firm size	7.1	25.1	6.8	-4.5
Spillovers $z^{spillover}$	0.889	0.815	0.893	0.5
Output (normalized)	1.000	1.110	0.981	-1.9
	Early	Late	γ^{team} only	Change
Inventor wage w	0.603	0.705	0.759	25.8
Prod. cutoff (exit)	1.004	1.257	1.312	30.6
Prod. cutoff (team)	1.579	1.710	1.525	-3.4
Share of solo-based firms	93.7	84.7	60.2	-33.5
Share of team patents	57.8	91.7	96.4	38.6
Employment share of firms with $n > 145$	32.8	79.2	79.9	47.0
Average firm size	7.1	25.1	30.4	328.9
Spillovers $z^{spillover}$	0.889	0.815	0.810	-8.9
Output (normalized)	1.000	1.110	1.189	18.9

Notes: This table reports the counterfactual exercises in which we vary one parameter at a time—$\bar{z}$ (top panel), x_{min}^{team} (middle panel), and γ^{team} (bottom panel)—while holding all other parameters fixed at their early-period values. The first two columns are the model moments for the early and late periods, respectively. The third column presents the model-implied value when only the given parameter is adjusted to its late-period estimate, and the last column reports the corresponding percent or percentage change relative to the early-period benchmark.

Share of team patents. A decline in solo productivity $\bar{z}$ can about half of the increase in the share of team patents by lowering the productivity cutoff at which firms choose to organize inventors into teams rather than having them work alone. In contrast, the decline in the minimum team productivity x_{min}^{team} has the opposite effect on the share of team patents because it raises the minimum scale needed for team production to be profitable. The increase in the upside potential of teams, captured by the thicker right tail of the team productivity distribution (lower γ^{team}), increases the predicted team patent share substantially, to 96% of all patents.

Inventor employment concentration. A decline in solo productivity $\bar{z}$ lowers the productivity cutoff for adopting team production. This in turn raises the average firm size slightly and results in a small increase in the employment share of large firms, accounting for about 10% of the increase in the data. The opposite is true for a decline in the minimum team productivity x_{min}^{team}. The increase in the right tail of team productivity draws (i.e., lower γ^{team}) generates a substantially larger rise in average firm size and the employment share of large firms than what we observe in the data.

Innovative output and knowledge spillovers. Declining solo and team productivity each reduce innovative output but have only minor effects on knowledge spillovers. Interestingly, declining minimum team productivity actually raises knowledge spillovers by reallocating inventors toward smaller firms. Yet this is not enough to combat the within-firm negative productivity shock for team-based firms induced by declining team productivity. The increase in the returns to scale benefit of using teams (declining γ^{team}) instead raises output but reduces spillovers since workers are reallocated toward larger firms, as we discuss more in the next section.

6.3 Effects on Misallocation

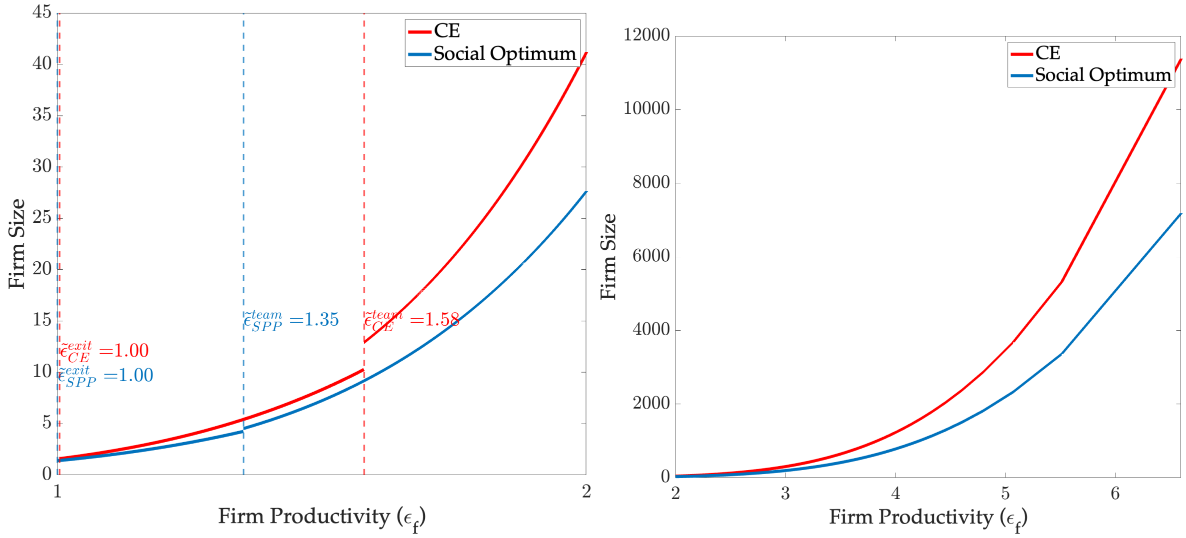
We next compare the competitive equilibrium allocations in the early and late periods with the corresponding social optimum given the different idea production technologies in the two periods. We extend Section 4 by allowing the planner to choose the mass of firms, firm entry and exit, and assign firms to use solo or team production. The full set-up is outlined in Appendix A.4. The parameter values are the same as in Table 2.

Firm size decision rules. First, we compare the firm-size decision rules under the competitive equilibrium and the social planner's allocation in both periods. Figure 5 illustrates the decision rules separately for the early and late periods. To enhance visibility, we split the figure into two subfigures that zoom in on the low- and high-productivity regions, shown in the left and right panels for each period. We find differences at both the extensive and intensive margins of inventor employment, as well as in the firm exit cutoffs.

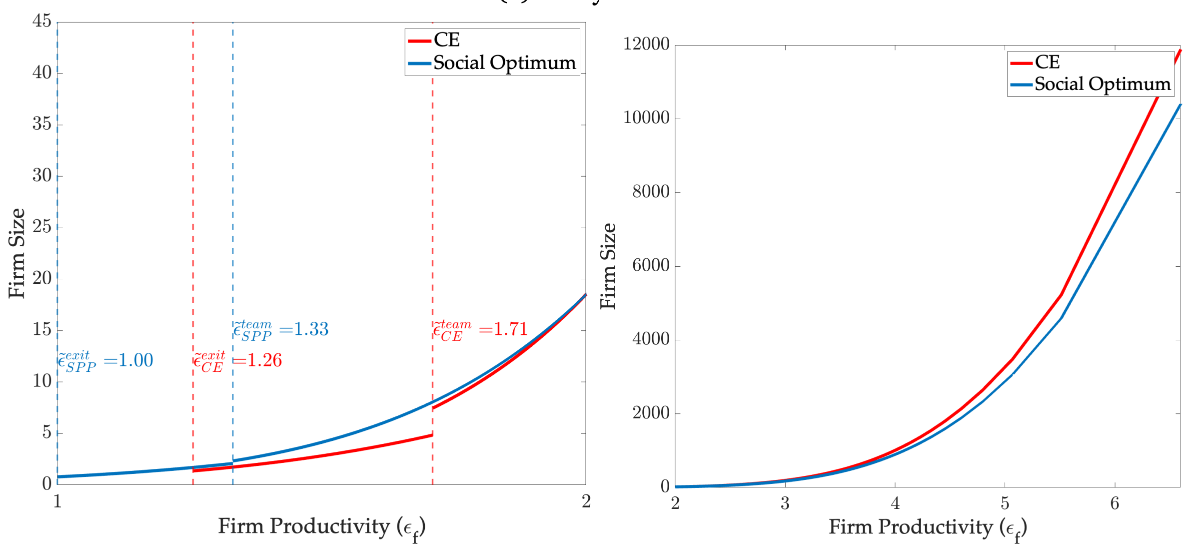
At the extensive margin, the planner chooses a lower productivity cutoff for adopting team production than in the competitive equilibrium in both periods ($\tilde{\varepsilon}^{team}_{SPP} < \tilde{\varepsilon}^{team}_{CE}$). This occurs because the planner internalizes the higher marginal contribution to aggregate output generated by team-based production.

At the intensive margin, the planner chooses a larger firm size for low productivity draws ε_f—particularly below the competitive equilibrium cutoff ($\tilde{\varepsilon}^{team}_{CE}$)—and a smaller firm size for high productivity draws—above that cutoff—relative to the competitive equilibrium. The planner attains higher aggregate output by internalizing knowledge spillovers and reallocating inventors toward smaller firms, whose marginal spillover effect is higher. This causes the planner to increase the size of smaller firms relative to the competitive equilibrium. In addition, for firms with productivity between the social planner's ($\tilde{\varepsilon}^{team}_{SPP}$) and the competitive equilibrium cutoffs ($\tilde{\varepsilon}^{team}_{CE}$), the planner adopts team production while firms remain in solo production under the competitive equilibrium because of their small size. This difference in organizational mode further contributes to the size gap between the planner's allocation and the competitive equilibrium. All of these results are present in both periods.

Lastly, the social planner sets the exit cutoff at the lower bound of the productivity distribution, $\tilde{\varepsilon}^{exit}_{SPP} = 1$, implying no firm entry nor exit. In contrast, the competitive equilibrium features a slightly higher exit cutoff, $\tilde{\varepsilon}^{exit}_{CE} = 1.0043$, which generates a positive entry and exit rate of 2.57%. As a result, the planner sustains a larger mass of



(a) Early Period



(b) Late Period

Figure 5: Firm Size Decision Rule: CE vs. SPP in the Early and Late Periods

Note: These figures illustrate firm-size decision rules under the competitive equilibrium and the social planner's allocation for the early period (top panel) and the late period (bottom panel). In each panel, the left subfigure focuses on the low-productivity (small-firm) region, while the right subfigure focuses on the high-productivity (large-firm) region to highlight the underlying patterns clearly. The red lines correspond to the competitive equilibrium, and the blue lines indicate the planner's equilibrium. The vertical lines denote two productivity cutoffs for adopting team production and firm exit in each equilibrium for each period.

firms than in the competitive equilibrium, $M_{SPP} = 0.1974 > M_{CE} = 0.1447$. Here, the planner internalizes the benefits of additional firms in the spillover term unlike entrants in the competitive equilibrium. Increasing firm mass allows the planner to spread labor across more firms to enhance spillovers given the concavity in the spillover.

Comparing the early and late periods, at the extensive margin, the productivity cutoff for adopting team production declines in the planner's equilibrium to take advantage of the greater upside potential of teams, while it rises in the competitive equilibrium because of the increase in the wage in competitive equilibrium. At the intensive margin, conditional on firm productivity, solo firms become smaller in the late period, while team-based firms become larger relative to the early period in both the competitive equilibrium and the planner's solution. However this pattern is more pronounced in the competitive equilibrium, particularly for solo firms. Thus, the competitive equilibrium amplifies the dispersion in firm size relative to the efficient allocation in the late period. These shifts are clearly illustrated in Figure B4 in the Appendix, where we plot the same series across periods within each competitive equilibrium and the social planner's equilibrium.

Firm-level wedges. Next, we compute the firm-level wedges τ_f, defined in (4.3) as the gap between the social marginal product and private marginal revenue product of inventor labor, evaluated at the competitive equilibrium allocation. Note that allowing endogenous choice between solo and team production creates a productivity cutoff in both the social planner's allocation and the competitive equilibrium. Also, endogenous firm entry and exit creates another productivity cutoff that drives firms to exit. We therefore evaluate the wedges accordingly by taking into account the optimal regime of production under each case.[14]

[14] Specifically, as in Figure 5, for firms below the social planner's team cutoff $\tilde{\varepsilon}^{team}_{SPP}$, both allocations involve solo production. For firms between the planner $\tilde{\varepsilon}^{team}_{SPP}$ and competitive equilibrium team cutoffs $\tilde{\varepsilon}^{team}_{CE}$, the planner adopts team production while the competitive equilibrium remains in solo production. For firms above the competitive equilibrium cutoff $\tilde{\varepsilon}^{team}_{CE}$, both allocations involve team production. Furthermore, there are no active firms below the competitive equilibrium exit cutoff, $\tilde{\varepsilon}^{\text{exit}}_{CE}$, unlike in the planner's allocation, so we cannot evaluate wedges for these firms.

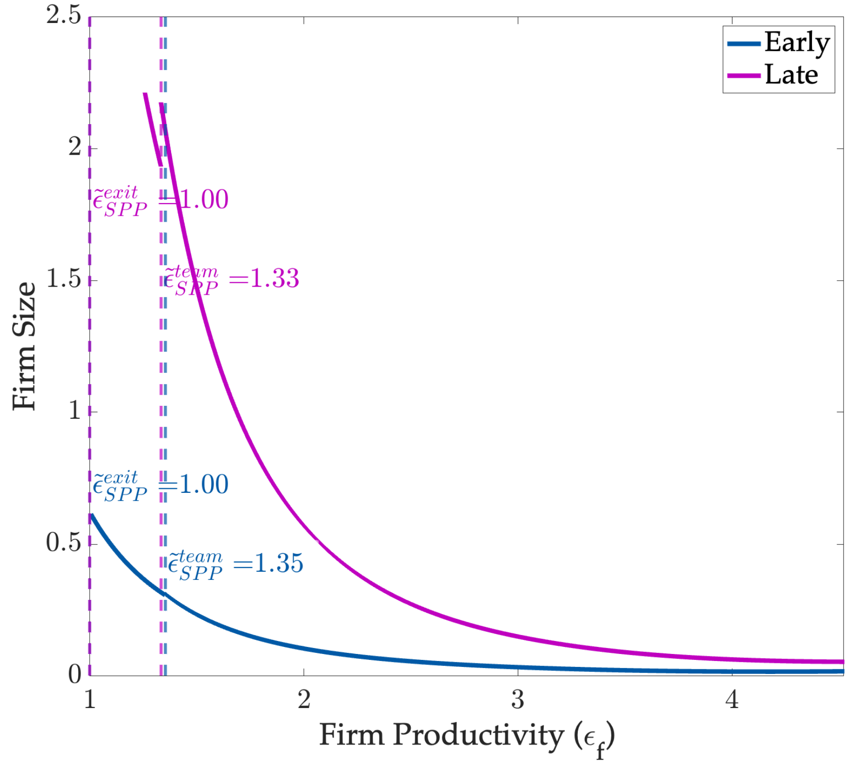


Figure 6: Firm-level Wedges: CE vs. SPP in the Early and Late Periods

Note: The figure illustrates the firm-level wedges τ_f between the competitive equilibrium and the social optimum, defined in equation (4.3), as a function of firm productivity ε_f. The blue lines correspond to the early period, and the purple lines correspond to the late period. The dashed vertical lines denote the two productivity cutoffs for adopting team production and firm exit in each period.

Figure 6 plots the wedges as a function of firm-level productivity, ε_f. In both periods, the wedges are decreasing in firm productivity (and therefore in firm size), consistent with the analytical results in Propositions 1 and 2. In addition, within each period, there is a modest increase in wedges around the social planner's team-production cutoff, $\tilde{\varepsilon}^{team}_{SPP}$. This pattern reflects differences at both the intensive and extensive margins between the competitive equilibrium and the planner's allocation.

At the intensive margin, as seen in Figure 5, there is a gap in firm size conditional on productivity under both solo and team production. In particular, the social planner selects a larger (smaller) size for smaller (larger) firms relative to the competitive equilibrium, which creates a source of wedges.

At the extensive margin, the competitive equilibrium features a higher cutoff for adopting team production than the planner, $\tilde{\varepsilon}^{team}_{CE} > \tilde{\varepsilon}^{team}_{SPP}$, in both periods. Consequently, there exists a productivity range between these cutoffs in which firms choose solo production in the competitive equilibrium even though team production is socially optimal. This discrepancy in organizational choice introduces an additional source of misallocation. Such a margin is absent in the simplified framework of Section 4.

Furthermore, misallocation arises from the difference in exit cutoffs between the competitive equilibrium and the planner's allocation. The planner choose a lower exit cutoff and thus assigns inventors to firms over a wider range of productivity, whereas firms exit earlier in the competitive equilibrium. As a result, there exists a range of productivity below the competitive equilibrium exit cutoff, $\tilde{\varepsilon}^{exit}_{CE}$, in which no firms operate under the competitive equilibrium but remain active under the planner. Because the competitive equilibrium assigns zero scale to these firms, wedges may become unbounded in this range. We therefore exclude this range in Figure 5 and truncate the figure at the competitive equilibrium exit cutoff in both periods.

Comparing wedges across the two periods, the level of wedges is uniformly higher in the late period across the entire productivity distribution. Moreover, the increase in wedges around the social planner's team-production cutoff is more pronounced in the late period, resulting in a steeper slope in that productivity range. This comes from the above mechanism through intensive and extensive margin differences. Lastly, as the competitive equilibrium exit cutoff rises in the late period, amplifying misallocation along the firm exit margin. Implementation of the first best would therefore involve larger R&D subsidies to small firms and/or a higher subsidy to entry in the later period.

Aggregate outcomes. Based on these results, we finally quantify the aggregate implications of the gap between the competitive equilibrium and the social planner's allocation in each period. Table 6 reports the results, measuring the deviation of the competitive equilibrium from the social optimum in aggregate innovative output, knowledge spillovers, and the employment share of large firms.

In the early period, innovative output in the competitive equilibrium is 8.35% below

Table 6: Competitive Equilibrium vs. Social Planner (CE to SPP)

Variable	Early	Late
Total Output	-8.35%	-9.63%
Spillovers ($z^{spillover}$)	-2.17%	-9.11%
Emp share of firms with $n > 145$	7.71pp	24.36pp

Notes: This table reports the deviation of the competitive equilibrium (CE) from the social planner's allocation (SPP) for aggregate innovative output, knowledge spillovers, and the employment share of large firms. The first column presents results for the early period, and the second column reports the counterparts for the late period.

the efficient allocation. In the late period, the gap widens to 9.63%. Changes in the idea production function therefore amplify misallocation by increasing the dispersion of wedges across firms. The endogenous decline in knowledge spillovers in the competitive equilibrium is a major driver of this widening gap, and the gap between the planner's realized spillovers and the competitive equilibrium's spillovers grows by a factor of four in the late period.

The output gains under the planner's allocation arise through two channels. First, the planner internalizes knowledge spillovers and retains a larger mass of firms, and second, the planner expands low-productivity firms and assigns them to team production when socially optimal. In the competitive equilibrium, the higher equilibrium wage induces excessive exit, resulting in a smaller mass of active firms, and prevents low-productivity firms from scaling up to the point at which team productivity exceeds solo productivity. The planner corrects this distortion by reallocating inventors and lowering the productivity threshold for adopting team production and firm exit.

Finally, the planner chooses a lower employment share of large firms than the competitive equilibrium in both periods, and this gap widens significantly in the late period. Note that, comparing the planner's allocations across time periods, the planner still chooses a much higher employment share of large firms in the late period compared to the early period. However, the increase under the planner is considerably smaller than the pronounced rise in employment concentration observed in the competitive equilibrium.

7 Conclusion

This paper studies how the rise of team-based idea production interacts with firm boundaries and shapes the allocation of innovative labor across firms and its aggregate implications. We document a dramatic shift toward team production in U.S. patenting and a concurrent rise in inventor employment concentration at large firms. We also provide evidence that firms play a central role in determining the set of possible collaborators and, therefore, the boundaries of innovative teams.

To interpret these patterns, we develop a tractable general equilibrium model in which team formation occurs within firms and knowledge spillovers operate across firms. We introduce a new microfoundation for team formation in which inventors draw heterogeneous bilateral productivities with potential collaborators, but only within firm boundaries. A larger pool of coworkers increases the likelihood of forming a highly productive match (e.g., exploiting complementarities across team members). As a result, team production raises returns to scale in inventor labor, influencing firm size, the allocation of inventors to firms, and aggregate innovative output and knowledge spillovers.

We estimate the model's idea production function using the U.S. patent data and show that team production increases returns to scale in inventor labor in the data, consistent with our new microfoundation. We further find that the idea production function has changed over time with a decline in average inventor productivity and a rise in the upside potential of teams. Our model predicts that these shifts in the idea production function jointly increase inventor employment concentration while endogenously reducing knowledge spillovers across firms.

The welfare implications of rising team production therefore depend critically on firm boundaries and the structure of knowledge spillovers. One promising direction for future research is to examine how the transition toward team-based idea production, particularly within a smaller set of innovative firms, shapes the long-run dynamics of knowledge accumulation for R&D workers.

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A Model Appendix

A.1 Empirical Support for Modeling Assumptions

One model assumption that deserves additional discussion is the assumption that there is no serial collaboration or team-specific human capital. Our model is a static, one period model, so there is no natural way of modeling accumulation of team-specific capital. Moreover, somewhat surprisingly, patenting teams are quite short lived in the data: 80% of teams in our sample only patent together once. Even for pairs of inventors within larger teams, 65% of pairs never collaborate again.

An illustrative example is inventor Adam Barth whose patenting history is as follows:

1. 2011, Google: "Automatically updating browser extensions, and applications thereof" w/ Johann Tomas Sigurdsson, Sigurdur Asgeirsson, Roger Tawa, Jeffrey Bailey
2. 2012, Google: "Method and system for browser identity" w/ Erik Kay and Aaron Boodman
3. 2013, Google: "Heterogeneous virtual machines sharing a security model" w/ Charles Reis
4. 2014, Alarm.com: "Force-sensitive occupancy sensing technology" w/ Mark Andrew Hanson, Samuel Alden Ridenour, Paul Michael Wempe
5. 2015, Alarm.com: "Fall detection and reporting technology" w/ Mark Andrew Hanson, Jean-Paul Martin, Christopher Silverman
6. 2017, Alarm.com: "Security system communicator and keypad device" w/ Zackary Watson, Daniel Todd Kerzner
7. 2020, Alarm.com: "Health and wellness management technology" w/ Mark Andrew Hanson, Christopher Silverman

At Google, a firm with a large pool of inventors, he had many distinct collaborations working on different problems. After moving to Alarm.com, a much smaller firm, he still worked with different co-inventors on different problems, but began to have some

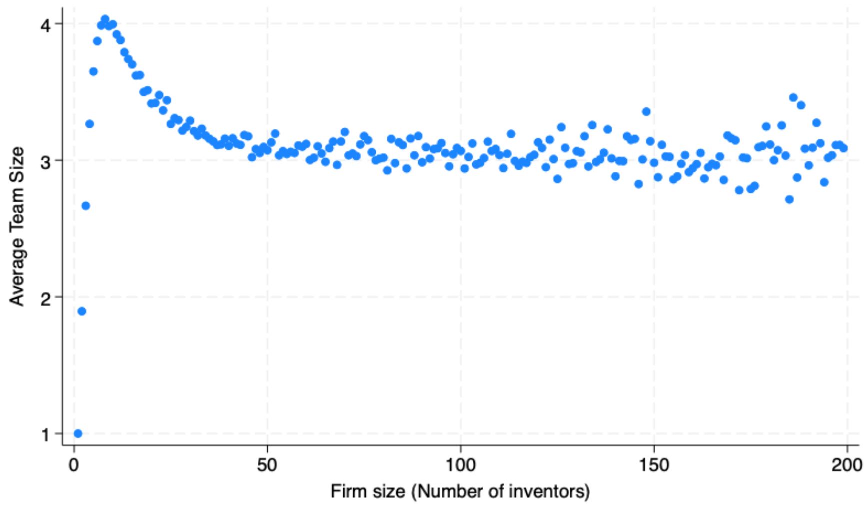


Figure A1: Average Team Size By Number of Inventors

Note: The figure illustrates the average team size within firms, as a function of the total number of patenting inventors at the firm in that year pooling all years in the USPTO data, 1976-2023.

repeated collaborations.

A second assumption is that inventors only work in teams of two. This assumption is stylized, since we also observe many teams of three to five inventors in the data (teams of two constitute 37% of team patents, and teams of less than 6 team members together constitute 80%). We expect a similar increasing, concave relationship between inventor productivity and the number of coworkers would hold in a richer model with productivity draws for larger groups of inventors. Importantly, the number of inventors per patent is not increasing one for one in the number of inventors at the firm (that is, firms with many inventors still divide them into smaller teams rather than pooling them into a single large team), leveling off at around three inventors (Figure A1).

In partial equilibrium the model predicts that output per worker is increasing in the number of coworkers. Figure A2, which plots patenting output per worker as a function of the number of (successful) inventors, shows that this is the case separately for two subperiods using USPTO data. Consistent with our finding that returns to scale increase, there is a level shift downward in output per worker in the later period. One concern with the figure is that output per worker looks highest for firms with one in-

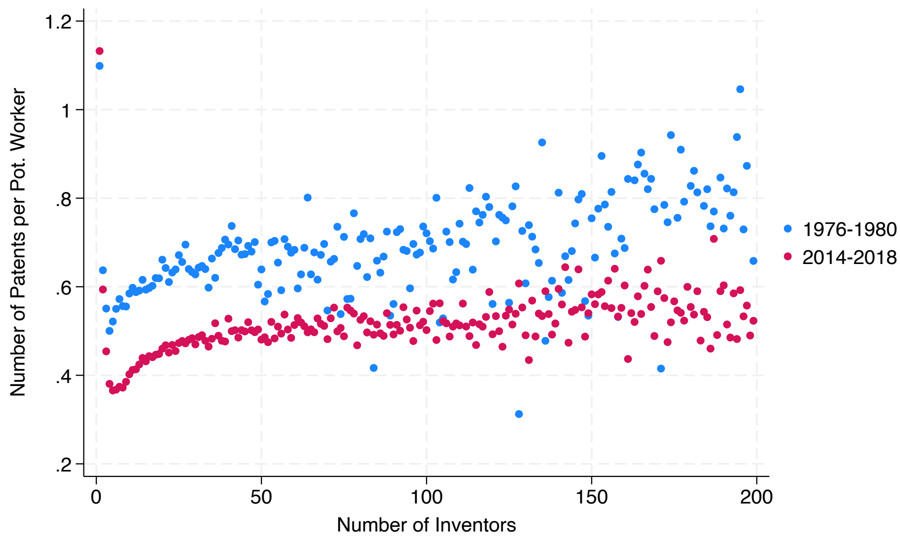


Figure A2: Number of patents per worker as a function of firm size in the USPTO data for two subperiods: 1976-1980 and 2014-2018.
Note: The figure illustrates the number of patents per worker within firms, as a function of the total number of patenting inventors at the firm in that year. The blue dots pool all years in the early period, 1976-1980, and the red dots pool all years in the late period, 2014-2018.

ventor. Recall that our measure of inventors is the number of successfully patenting inventors, which mechanically ensures that output per worker for single-inventor firms is at least one. Figure A3 corrects for this using employment histories of inventors from Revelio Labs to get a more accurate measure of inventors per firm (2008-2018), enabling us to include firm-year pairs with inventors but no patents, and shows that output per worker at single inventor firms is actually quite low. Otherwise the pattern is qualitatively similar to the USPTO figure.

Finally, the model predicts that larger firms will have a higher share of team patents in total patents. Defining firm size by the number of inventors, we estimate the following regression to examine the relationship between firm size and the propensity to engage in team patenting:

$$TeamPatShare_{ft} = \eta_f + \gamma_t + \log(Num.Inventors)_{ft} + \epsilon_{ft} \tag{A.1}$$

where $TeamPatShare_{ft}$ is the share of team patents out of total patents assigned by

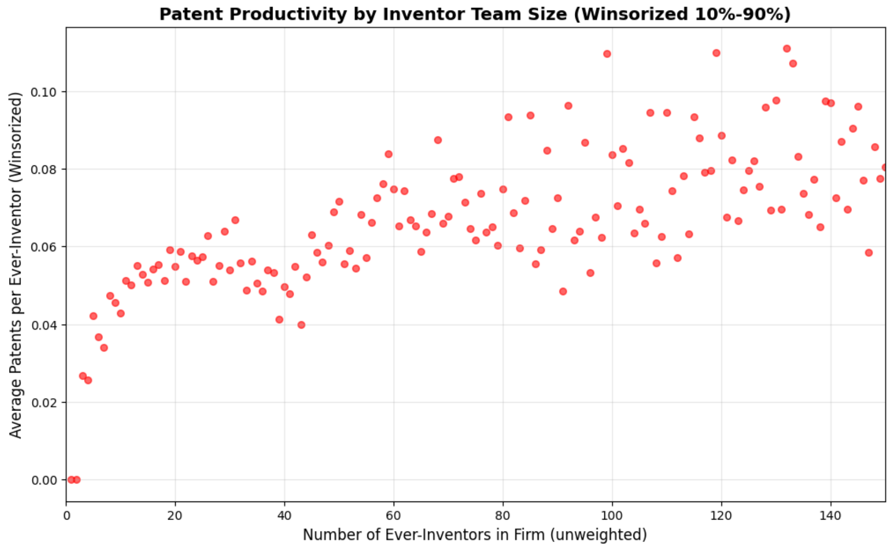


Figure A3: Number of patents per worker as a function of firm size in the Revelio Labs data, 2008-2018.

Note: The figure illustrates the number of patents per worker within firms, as a function of the total number of inventors at the firm in that year. The data is sourced from the Revelio Labs data, pooling all years between 2008 and 2018.

firm f in year t; $\log(Num.Inventors)_{ft}$ is the log number of inventors within firm f in t; and η_f and γ_t are firm and year fixed effects, respectively.

Table A1 presents the results, where the first column uses all USPTO assignees, and the second column restricts the sample to public firms in Compustat. Across both samples, we find that larger firms exhibit a higher share of team patents than smaller firms. The effect is economically meaningful: a 1% increase in the number of inventors within a firm is associated with a 20.4 pp increase in the share of team patents for USPTO firms, and a 12 pp for public firms.

A.2 Derivation of Team Productivity

A.2.1 Discrete Firm Size

Recall that bilateral productivity is:

$$z_{ijt} \sim P(x_{min}^{team}, \gamma^{team}), \quad P(z \leq x) = 1 - (\frac{x_{min}^{team}}{x})^{\gamma^{team}} \quad \forall x \geq x_{min}^{team}$$

Table A1: Team Patent Share and Firm Size

	(1) All Firms	(2) Public Firms
Log num. inventors	0.204***	0.122***
	(282.52)	(22.46)
N of Obs.	717,111	74,501
Firm FE	Yes	Yes
Year FE	Yes	Yes
Adj. R^2	0.491	0.375

Note: The table presents the regression result from equation (A.1), where the dependent variable is the share of team patents within a firm. The coefficient of interest is on the log number of inventors in the firm. Firm and year fixed effects are included, respectively. Column (1) covers all USPTO assignees, and Column (2) includes Compustat firms only. *** p<0.01, ** p<0.05, * p<0.1

Let the max of n draws be

$$M_n \equiv \max\{z_{1jt}, z_{2jt}, .., z_{njt}\}$$

then

$$F_{M_n}(x) \equiv P(M_n \leq x) = \Big[1 - (\frac{x_{min}^{team}}{x})^{\gamma^{team}}\Big]^n \quad \forall x \geq x_{min}^{team}$$

Then, we can derive the pdf as follows:

$$f_{M_n}(x) = n\gamma^{team}(\frac{(x_{min}^{team})^{\gamma^{team}}}{x^{\gamma^{team}+1}})\Big[1 - (\frac{x_{min}^{team}}{x})^{\gamma^{team}}\Big]^{n-1} \quad \forall x \geq x_{min}^{team}.$$

Thus, the expected value is

$$\mathbb{E}(M_n) = \int_{x_{min}^{team}} x f_{M_n}(x)dx = x_{min}^{team} nB(n, 1 - \frac{1}{\gamma^{team}}),$$

where B is Beta function.[15]

[15]It can be derived with a transformation of variable with $t = 1 - (x_{min}^{team}/x)^{\gamma^{team}}$. So that it can be rephrased as $\mathbb{E}(M_n) = nx_{min}^{team} \int_0^1 t^{n-1}(1-t)^{-\frac{1}{\gamma^{team}}} dt$.

Note that for large n, the Beta function asymptotically becomes:

$$B(n, 1-\frac{1}{\gamma^{team}}) = \frac{\Gamma(n)\Gamma(1-\frac{1}{\gamma^{team}})}{\Gamma(n+1-\frac{1}{\gamma^{team}})} \simeq \Gamma(1-\frac{1}{\gamma^{team}})n^{-1+\frac{1}{\gamma^{team}}}.$$

Therefore, the expected productivity of working in a firm with n team members becomes:

$$\mathbb{E}(M_n) = x_{min}^{team}\Gamma(1-\frac{1}{\gamma^{team}})n^{\frac{1}{\gamma^{team}}} \tag{A.2}$$

which increases in n.

A.2.2 Continuous Firm Size

Assume that worker bilateral productivity is drawn from an inhomogeneous Poisson point process with intensity function within firm of size $n \in [0, \infty)$.

$$\lambda(x) = n\frac{f(x)}{F(x)}$$

Then, the distribution function of the maximum talent is

$$\begin{aligned} F_{z_{it}^{team}}(x) &= \mathbb{P}[N(x, +\infty) = 0] \\ &= \exp\left(-\int_x^{+\infty} \lambda(u)du\right) \\ &= F_{z_{it}}(x)^n, \end{aligned}$$

and thus, everything follows the same as before.

A.3 Proofs for Propositions

A.3.1 Proof for Proposition 1

We evaluate wedges in equation (4.6) at competitive equilibrium under solo production. Note that the firm decision rule in competitive equilibrium remains the same as in (3.9), except that the equilibrium wage is determined by the labor market clearing with solo production. The equilibrium wage is pinned down by

$$\int n^{solo}(\varepsilon_f)g(\varepsilon_f) = \bar{N},$$

which gives

$$w = \frac{1}{\bar{N}^{1-\alpha}}\bar{z}^{spillover}\alpha\bar{z}x^f_{min}\left(\frac{\gamma^f}{\gamma^f - \frac{1}{1-\alpha}}\right)^{1-\alpha}.$$

We normalize $\bar{N} = 1$, and this gives a closed-from solution for firm labor demand under solo production as follows:

$$n^{solo}_f = \frac{(\gamma^f - \frac{1}{1-\alpha})\varepsilon_f^{\frac{1}{1-\alpha}}}{\gamma^f(x^f_{min})^{\frac{1}{1-\alpha}}}.$$

This also determines spillovers and total output at the equilibrium,

$$\bar{z}^{spillover} = (\bar{z}x^f_{min})^{(1-\beta)\theta}\left(\frac{\gamma^f}{\gamma^f - (\frac{1-\alpha+\alpha\beta}{1-\alpha})}\right)^{\theta}\left(\frac{\gamma^f - \frac{1}{1-\alpha}}{\gamma^f}\right)^{\beta\theta}$$

$$Y = \bar{z}^{spillover}\bar{z}x^f_{min}\left(\frac{\gamma^f}{\gamma^f - \frac{1}{1-\alpha}}\right)^{1-\alpha}.$$

Plugging them into (4.6), we obtain the following expression for wedges under solo production:

$$\tau^{solo}_f = \frac{\theta\beta}{\alpha}(x^f_{min})^{-\frac{\alpha(\beta-1)}{1-\alpha}}\left(\frac{\gamma^f - \frac{1-\alpha+\alpha\beta}{1-\alpha}}{\gamma^f - \frac{1}{1-\alpha}}\right)\varepsilon_f^{\frac{\alpha(\beta-1)}{1-\alpha}}. \tag{A.3}$$

Since $\beta < 1$ and $\alpha \in (0,1)$, (A.3) is decreasing in firm productivity—and hence in employment size, given their positive relationship in competitive equilibrium.

A.3.2 Proof for Proposition 2

We evaluate wedges in equation (4.9) at competitive equilibrium under team production. As before, the equilibrium wage is pinned down by the following, which determines firm decision rule in (3.9) under team production:

$$\int n^{team}(\varepsilon_f)g(\varepsilon_f) = \bar{N},$$

which gives

$$w_t = \bar{z}_t^{spillover}(\frac{1}{\gamma^{team}} + \alpha)x_{min}^{team}\Gamma(1 - \frac{1}{\gamma^{team}})x_{min}^{f}\left(\frac{\gamma^f}{\gamma^f - \frac{1}{1-(\frac{1}{\gamma^{team}}+\alpha)}}\right)^{\frac{1}{1-(\frac{1}{\gamma^{team}}+\alpha)}}.$$

Thus, firm labor demand under team production is

$$n_f^{team} = \frac{(\gamma^f - \frac{1}{1-(\frac{1}{\gamma^{team}}+\alpha)})\varepsilon_f^{\frac{1}{1-(\frac{1}{\gamma^{team}}+\alpha)}}}{\gamma^f(x_{min}^f)^{\frac{1}{1-(\frac{1}{\gamma^{team}}+\alpha)}}}.$$

This also determines spillovers and total output at the equilibrium,

$$\begin{aligned}\bar{z}^{spillover} =& \left(x_{min}^{team}\Gamma(1 - \frac{1}{\gamma^{team}})x_{min}^{f}\right)^{(1-\beta)\theta}\left(\frac{\gamma^f}{\gamma^f - \frac{1-\alpha+\alpha\beta}{1-(\frac{1}{\gamma^{team}}+\alpha)}}\right)^{\theta} \\ &\times\left(\frac{\gamma^f - \frac{1}{1-(\frac{1}{\gamma^{team}}+\alpha)}}{\gamma^f}\right)^{((1-\beta)\frac{1}{\gamma^{team}}+\beta)\theta}\end{aligned}$$

$$Y = \bar{z}^{spillover}x_{min}^{team}\Gamma(1 - \frac{1}{\gamma^{team}})x_{min}^{f}\left(\frac{\gamma^f}{\gamma^f - \frac{1}{1-(\frac{1}{\gamma^{team}}+\alpha)}}\right)^{1-(\frac{1}{\gamma^{team}}+\alpha)}.$$

Plugging them into (4.9), we obtain the following expression for wedges under team production:

$$\tau_f^{team} = \frac{\theta\left(\beta + (1-\beta)\frac{1}{\gamma^{team}}\right)}{\alpha + \frac{1}{\gamma^{team}}}(x_{min}^f)^{-\frac{\alpha(1-\beta)}{1-(\alpha+\frac{1}{\gamma^{team}})}}\left(\frac{\gamma^f - \frac{1-\alpha+\alpha\beta}{1-(\frac{1}{\gamma^{team}}+\alpha)}}{\gamma^f - \frac{1}{1-(\alpha+\frac{1}{\gamma^{team}})}}\right)\varepsilon_f^{\frac{\alpha(1-\beta)}{1-(\alpha+\frac{1}{\gamma^{team}})}}. \quad \text{(A.4)}$$

If $\beta < 1$ and $\alpha + \frac{1}{\gamma^{team}} < 1$, (A.4) is decreasing in firm productivity—and hence in employment size, given their positive relationship in competitive equilibrium.

A.3.3 Proof for Proposition 3

Comparing (A.3) with (A.4), we find that the wedges can be written in a uniform form as follows:

$$\tau_f = \frac{\theta(\beta+(1-\beta)\eta)}{\alpha+\eta}\left(\frac{\gamma^f - \frac{1-\alpha+\alpha\beta}{1-\alpha-\eta}}{\gamma^f - \frac{1}{1-\alpha-\eta}}\right)\left(\frac{\varepsilon_f}{x^f_{min}}\right)^{-\frac{\alpha(1-\beta)}{1-\alpha-\eta}}.$$

When $\eta = 0$, this gives the solo wedge; when $\eta = \frac{1}{\gamma^{team}}$, this gives the team wedge. Therefore, to compare wedges in both economies, it suffices to check the sign of the following derivative:

$$\frac{\partial \log \tau_f}{\partial \epsilon_f} = \frac{(1-\beta)\alpha-\beta}{((1-\beta)\eta+\beta)(\alpha+\eta)} + \frac{\alpha(1-\beta)}{(1-\alpha-\eta)^2}\log\left(\frac{\varepsilon_f}{x^f_{\text{min}}}\right) + \frac{\gamma^f\alpha(1-\beta)}{((1-\alpha-\eta)\gamma^f-(1-\alpha+\alpha\beta))((1-\alpha-\eta)\gamma^f-1)} > 0.$$

Since this is positive for $\eta \in (0, \frac{1}{\gamma^{team}})$ when $0 < \beta < \frac{\beta}{1-\beta} < \alpha < \alpha + \frac{1}{\gamma^{team}} < 1$, the wedges are also larger in team economy.

A.4 The Social Planner's Problem with Endogenous Firm Mass

This section presents the full-fledged setup of the social planner's problem with endogenous firm entry and exit. The following shows the planner's maximization:

$$\max_{M,A,n_f,\text{type}_f} M\left(\int_A (\bar{z}^{\text{spillover}} z_f n_f^\alpha - c_f)df - c_e\right)$$

subject to

$$M\int_A n_f df = \bar{N},\ \bar{z}^{\text{spillover}} = \left(M\int_A z_f^{1-\beta} n_f^\beta df\right)^\theta,$$

where M denotes the mass of firms, A denotes the set of active firms (equivalently, the choice of an exit cutoff), and type_f indicates whether firm f operates as a team or as a solo firm. The planner maximizes aggregate output net of fixed costs, accounting for both the operating cost c_f and the entry cost c_e, which are paid in units of the consumption good.

First, the social planner chooses the mass of firms M and incurs a fixed entry cost c_e per firm. After entry, each firm f draws an idiosyncratic productivity shock ε_f. Conditional on ε_f, the planner assigns to each firm f a productivity type—solo or team—with productivity given by

$$\begin{aligned} z_f^{solo} &= \bar{z}\varepsilon_f, \\ z_f^{team} &= x_{\min}^{team}\Gamma\left(1-\frac{1}{\gamma^{team}}\right) n_f^{\frac{1}{\gamma^{team}}}\varepsilon_f, \end{aligned}$$

and chooses its labor input n_f. The planner can instead shut down the firm. Each active firm $f \in A$ incurs a fixed cost c_f.

Let λ denote the Lagrange multiplier on the labor market clearing constraint. The Lagrangian associated with the social planner's problem is given by

$$\mathcal{L} = M^{1+\theta}\left(\int_A z_f^{1-\beta} n_f^{\beta} df\right)^{\theta}\left(\int_A z_f n_f^{\alpha} df\right) - Mc_f\left(\int_A df\right) - Mc_e - \lambda M\left(\int_A n_f df\right) + \lambda\bar{N}.$$

For expositional simplicity, let

$$Z^{raw} = \int_A z_f^{1-\beta} n_f^{\beta} df,\ Y^{raw} = \int_A z_f n_f^{\alpha} df,\ Y = M^{1+\theta}(Z^{raw})^{\theta}(Y^{raw}).$$

The first-order condition for a solo firm's labor choice is given by

$$\frac{Y}{M}\left(\theta\beta\frac{(z_f^{solo})^{1-\beta}(n_f^{solo})^{\beta-1}}{Z^{raw}} + \alpha\frac{(z_f^{solo})(n_f^{solo})^{\alpha-1}}{Y^{raw}}\right) = \lambda. \tag{A.5}$$

The first-order condition for a team firm's labor choice is

$$\frac{Y}{M}\left(\theta\left(\beta+\frac{1-\beta}{\gamma^{team}}\right)\frac{(z_f^{team})^{1-\beta}(n_f^{team})^{\beta-1}}{Z^{raw}}+\left(\alpha+\frac{1}{\gamma^{team}}\right)\frac{(z_f^{team})(n_f^{team})^{\alpha-1}}{Y^{raw}}\right)=\lambda. \tag{A.6}$$

The marginal benefits from assigning firms to each production type are given by the optimized surplus under each technology as follows:

$$\begin{cases}\frac{Y}{M}\left(\theta\frac{(z_f^{solo})^{1-\beta}(n_f^{solo})^{\beta}}{Z^{raw}}+\frac{(z_f^{solo})(n_f^{solo})^{\alpha}}{Y^{raw}}\right)-c_f-\lambda n_f^{solo}, & \text{for solo firms,}\\ \frac{Y}{M}\left(\theta\frac{(z_f^{team})^{1-\beta}(n_f^{team})^{\beta}}{Z^{raw}}+\frac{(z_f^{team})(n_f^{team})^{\alpha}}{Y^{raw}}\right)-c_f-\lambda n_f^{team}, & \text{for team firms,}\\ 0, & \text{for inactive firms.}\end{cases} \tag{A.7}$$

The social planner assigns firm f to the production type that yields the highest marginal benefit.

Finally, the first-order condition for choosing the number of firms is

$$(1+\theta)\frac{Y}{M}-c_f\left(\int_A df\right)-c_e-\lambda\left(\int_A n_f df\right)=0. \tag{A.8}$$

The solution to the social planner's problem is jointly characterized by conditions (A.5)–(A.8) together with the labor market clearing condition.

The derivation of (A.7) is as follows. For simplicity, consider the following prototype problem:

$$J=\theta\log\int f(c,\varepsilon)d\varepsilon+\log\int g(c,\varepsilon)d\varepsilon,\ \ c(\varepsilon)\in\{0,1\}.$$

Consider a (measurable) perturbation $\delta c(\varepsilon)\in\{-1,0,1\}$ to the choice of firm types such that $\int |\delta c(\varepsilon)| d\varepsilon$ is small. Let $\Delta f(\varepsilon)$ denote the difference $f(1,\varepsilon) - f(0,\varepsilon)$. Then,

$$\delta \int f(c,\varepsilon) d\varepsilon = \int \Delta f(\varepsilon) \delta c(\varepsilon) d\varepsilon.$$

This is also small when $\Delta f(\varepsilon)$ is bounded. Thus,

$$\delta \log \int f(c,\varepsilon) d\varepsilon \doteq \frac{\int \Delta f(\varepsilon) \delta c(\varepsilon) d\varepsilon}{\int f(c,\varepsilon) d\varepsilon}.$$

Similar things hold for g. Therefore, the first-order difference is

$$\begin{aligned} \delta J &\doteq \theta \frac{\int \Delta f(\varepsilon) \delta c(\varepsilon) d\varepsilon}{\int f(c,\varepsilon) d\varepsilon} + \frac{\int \Delta g(\varepsilon) \delta c(\varepsilon) d\varepsilon}{\int g(c,\varepsilon) \varepsilon} \\ &= \int \left(\theta \frac{\Delta f(\varepsilon)}{\int f(c,\varepsilon) d\varepsilon} + \frac{\Delta g(\varepsilon)}{\int g(c,\varepsilon) d\varepsilon} \right) \delta c(\varepsilon) d\varepsilon. \end{aligned}$$

By the standard argument, this difference is necessarily zero for all valid $\delta c(\varepsilon)$ around the optimal choice of firm types. Finally, the first-order condition for optimality is

$$\theta \frac{\Delta f(\varepsilon)}{\int f(c,\varepsilon) d\varepsilon} + \frac{\Delta g(\varepsilon)}{\int g(c,\varepsilon) d\varepsilon} = 0.$$

Substituting into the expressions for f and g yields (A.7). Intuitively, the marginal contribution of changing a firm's type is given by a weighted sum of its effects on the spillover component and the productivity component.

Then, we can prove the following proposition, suggesting the existence of the two productivity cutoffs in the planner's equilibrium.

Proposition A4. *Suppose that* $0 < \beta < \beta + \frac{1-\beta}{\gamma^{team}} < 1$, $0 < \alpha < \alpha + \frac{1}{\gamma^{team}} < 1$, *and* $\frac{\beta}{1-\beta} < \alpha$. *When inactive, solo, and team firms coexist, there exist cutoffs* $\tilde{\varepsilon}^{solo}_{spp}$ *and* $\tilde{\varepsilon}^{team}_{spp}$ *such that*

$$x^f_{\min} < \tilde{\varepsilon}^{solo}_{spp} < \tilde{\varepsilon}^{team}_{spp}.$$

Firms are inactive if $\varepsilon_f < \tilde{\varepsilon}^{solo}_{spp}$, *operate as solo firms if* $\tilde{\varepsilon}^{solo}_{spp} \leq \varepsilon_f < \tilde{\varepsilon}^{team}_{spp}$, *and operate as* *team firms if* $\varepsilon_f \geq \tilde{\varepsilon}^{team}_{spp}$*. Moreover, at the solo–team cutoff* $\tilde{\varepsilon}^{team}_{spp}$*, we have*

$$z_f^{solo} < z_f^{team} \quad \text{and} \quad n_f^{solo} < n_f^{team}.$$

Proof. Fix λ, Y, M, Z^{raw}, and Y^{raw}. Equations (A.5) and (A.6) imply that n^{solo} and n^{team} are continuously differentiable and strictly increasing in ε. The same holds for z^{solo} and z^{team}, and therefore for the marginal benefit functions MB^{solo} and MB^{team} in (A.7). Hence, at the solo–team cutoff $\tilde{\varepsilon}^{team}_{\text{spp}}$, the planner must be indifferent between assigning the firm to solo or team production, which requires $MB^{solo}(\tilde{\varepsilon}^{team}_{spp}) = MB^{team}(\tilde{\varepsilon}^{team}_{spp})$.

First, we show that $z_f^{solo} < z_f^{team}$ and $n_f^{solo} < n_f^{team}$ at the solo–team cutoff $\tilde{\varepsilon}^{team}_{\text{spp}}$. Let μ denote the common marginal benefit of assigning a firm to solo and team production at this cutoff. Then (z_f^{solo}, n_f^{solo}) and (z_f^{team}, n_f^{team}) solve, respectively, the same system of equations evaluated at $\eta = 0$ and $\eta = \frac{1}{\gamma^{team}}$:

$$\frac{Y}{M}\left(\theta(\beta + (1-\beta)\eta)\frac{z^{1-\beta}n^{\beta-1}}{Z^{raw}} + (\alpha+\eta)\frac{zn^{\alpha-1}}{Y^{raw}}\right) = \lambda,$$
$$\frac{Y}{M}\left(\theta(1-\beta-(1-\beta)\eta)\frac{z^{1-\beta}n^{\beta}}{Z^{raw}} + (1-\alpha-\eta)\frac{zn^{\alpha}}{Y^{raw}}\right) = c_f + \mu.$$

The second equation is obtained by substituting (A.5) and (A.6) into the marginal benefit expressions in (A.7). Differentiating this system with respect to η yields

$$\beta_\eta \Psi_Z\left((1-\beta)\frac{z'}{z} + (\beta-1)\frac{n'}{n}\right) + \alpha_\eta \Psi_Y\left(\frac{z'}{z} + (\alpha-1)\frac{n'}{n}\right) = -(1-\beta)\Psi_Z - \Psi_Y,$$
$$(1-\beta_\eta)\Psi_Z\left((1-\beta)\frac{z'}{z} + \beta\frac{n'}{n}\right) + (1-\alpha_\eta)\Psi_Y\left(\frac{z'}{z} + \alpha\frac{n'}{n}\right) = (1-\beta)\Psi_Z + \Psi_Y,$$

where,

$$\Psi_Z = \frac{Y}{M}\theta\frac{z^{1-\beta}n^{\beta}}{Z^{raw}},\ \Psi_Y = \frac{Y}{M}\frac{zn^{\alpha}}{Y^{raw}},\ \beta_\eta = \beta + (1-\beta)\eta,\ \alpha_\eta = \alpha + \eta,\ z' = \frac{dz}{d\eta},\ n' = \frac{dn}{d\eta}.$$

By Cramer's rule, we find that

$$\frac{z'}{z} = \frac{\eta}{D}((1-\beta)\Psi_Z + \Psi_Y)^2 > 0,$$
$$\frac{n'}{n} = \frac{1}{D}((1-\beta)\Psi_Z + \Psi_Y)^2 > 0,$$

where, the determinant is given by

$$D = (\beta_\eta(1-\beta)\Psi_Z + \alpha_\eta\Psi_Y)((1-\beta_\eta)\beta\Psi_Z + (1-\alpha_\eta)\alpha\Psi_Y) + ((1-\beta_\eta)(1-\beta)\Psi_Z + (1-\alpha_\eta)\Psi_Y)(\beta_\eta(1-\beta)\Psi_Z + (1-\alpha_\eta)\Psi_Y) > 0.$$

Therefore, $z_f^{solo} < z_f^{team}$ and $n_f^{solo} < n_f^{team}$.

Next, we show that the solo-versus-team cutoff $\tilde{\varepsilon}_{spp}^{team}$ is unique whenever it exists, i.e., MB^{solo} and MB^{team} can cross at most once. A sufficient condition is that whenever $MB^{solo}(\tilde{\varepsilon}_{spp}^{team}) = MB^{team}(\tilde{\varepsilon}_{spp}^{team})$, it holds that

$$\left.\frac{dMB^{solo}}{d\varepsilon}\right|_{\varepsilon=\tilde{\varepsilon}_{spp}^{team}} < \left.\frac{dMB^{team}}{d\varepsilon}\right|_{\varepsilon=\tilde{\varepsilon}_{spp}^{team}}.$$

We establish this by directly evaluating the derivatives at the cutoff $\tilde{\varepsilon}_{spp}^{team}$. The derivatives are, respectively,

$$\begin{aligned}
\frac{dMB^{solo}}{d\varepsilon} &= \left(\frac{Y}{M}\left(\theta\beta\frac{(z^{solo})^{1-\beta}(n^{solo})^{\beta-1}}{Z^{raw}} + \alpha\frac{(z^{solo})(n^{solo})^{\alpha-1}}{Y^{raw}}\right) - \lambda\right)\frac{dn^{solo}}{d\varepsilon} \\
&\quad + \frac{1}{\varepsilon}\frac{Y}{M}\left(\theta(1-\beta)\frac{(z^{solo})^{1-\beta}(n^{solo})^{\beta}}{Z^{raw}} + \frac{(z^{solo})(n^{solo})^{\alpha}}{Y^{raw}}\right) \\
&= \frac{1}{\varepsilon}\frac{Y}{M}\left(\theta(1-\beta)\frac{(z^{solo})^{1-\beta}(n^{solo})^{\beta}}{Z^{raw}} + \frac{(z^{solo})(n^{solo})^{\alpha}}{Y^{raw}}\right), \\
\frac{dMB^{team}}{d\varepsilon} &= \left(\frac{Y}{M}\left(\theta\left(\beta + \frac{1-\beta}{\gamma^{team}}\right)\frac{(z^{team})^{1-\beta}(n^{team})^{\beta-1}}{Z^{raw}} + \left(\alpha + \frac{1}{\gamma^{team}}\right)\frac{(z^{team})(n^{team})^{\alpha-1}}{Y^{raw}}\right) - \lambda\right)\frac{dn^{team}}{d\varepsilon} \\
&\quad + \frac{1}{\varepsilon}\frac{Y}{M}\left(\theta(1-\beta)\frac{(z^{team})^{1-\beta}(n^{team})^{\beta}}{Z^{raw}} + \frac{(z^{team})(n^{team})^{\alpha}}{Y^{raw}}\right) \\
&= \frac{1}{\varepsilon}\frac{Y}{M}\left(\theta(1-\beta)\frac{(z^{team})^{1-\beta}(n^{team})^{\beta}}{Z^{raw}} + \frac{(z^{team})(n^{team})^{\alpha}}{Y^{raw}}\right).
\end{aligned}$$

Here, we substitute (A.5) and (A.6) and cancel the first terms. At the cutoff $\tilde{\varepsilon}_{spp}^{team}$, the marginal benefits coincide. Let μ denote the common value, i.e.,

$$\begin{aligned}\mu &= \frac{Y}{M}\left(\theta\frac{(z_f^{solo})^{1-\beta}(n_f^{solo})^{\beta}}{Z^{raw}} + \frac{(z_f^{solo})(n_f^{solo})^{\alpha}}{Y^{raw}}\right) - c_f - \lambda n_f^{solo} \\ &= \frac{Y}{M}\left(\theta\frac{(z_f^{team})^{1-\beta}(n_f^{team})^{\beta}}{Z^{raw}} + \frac{(z_f^{team})(n_f^{team})^{\alpha}}{Y^{raw}}\right) - c_f - \lambda n_f^{team}.\end{aligned}$$

Therefore, evaluating the derivatives at the cutoff $\tilde{\varepsilon}_{spp}^{team}$, we obtain the following:

$$\begin{aligned}\left.\frac{dMB^{solo}}{d\varepsilon}\right|_{\varepsilon=\tilde{\varepsilon}_{spp}^{team}} &= \frac{1}{\varepsilon}\left(\beta\frac{Y}{M}\frac{(z_f^{solo})(n_f^{solo})^{\alpha}}{Y^{raw}} + (1-\beta)(\mu + c_f + \lambda n_f^{solo})\right), \\ \left.\frac{dMB^{team}}{d\varepsilon}\right|_{\varepsilon=\tilde{\varepsilon}_{spp}^{team}} &= \frac{1}{\varepsilon}\left(\beta\frac{Y}{M}\frac{(z_f^{team})(n_f^{team})^{\alpha}}{Y^{raw}} + (1-\beta)(\mu + c_f + \lambda n_f^{team})\right).\end{aligned}$$

Since at this cutoff $z_f^{solo} < z_f^{team}$ and $n_f^{solo} < n_f^{team}$, it follows that

$$\left.\frac{dMB^{solo}}{d\varepsilon}\right|_{\varepsilon=\tilde{\varepsilon}_{spp}^{team}} < \left.\frac{dMB^{team}}{d\varepsilon}\right|_{\varepsilon=\tilde{\varepsilon}_{spp}^{team}}.$$

As previously shown, this implies that the marginal benefit functions MB^{solo} and MB^{team} can cross at most once. Therefore, the solo–team cutoff is unique whenever it exists.

Finally, we show that the exit cutoff $\tilde{\varepsilon}_{spp}^{solo}$ is unique whenever it exists. Substituting (A.5) and (A.6) into the marginal benefit expressions in (A.7) implies that

$$\begin{aligned}MB_f^{solo} &= \frac{Y}{M}\left(\theta\,(1-\beta)\,\frac{(z_f^{solo})^{1-\beta}(n_f^{solo})^{\beta}}{Z^{raw}} + (1-\alpha)\,\frac{(z_f^{solo})(n_f^{solo})^{\alpha}}{Y^{raw}}\right) - c_f, \\ MB_f^{team} &= \frac{Y}{M}\left(\theta\left(1-\beta-\frac{1-\beta}{\gamma^{team}}\right)\frac{(z_f^{team})^{1-\beta}(n_f^{team})^{\beta}}{Z^{raw}} + \left(1-\alpha-\frac{1}{\gamma^{team}}\right)\frac{(z_f^{team})(n_f^{team})^{\alpha}}{Y^{raw}}\right) - c_f.\end{aligned}$$

Regarding both as a function of ε_f, each is strictly increasing in ε_f. Hence, their pointwise maximum can cross the horizontal line $MB = 0$ at most once. Therefore, the exit cutoff is unique whenever it exists.

□

Table B2: Model Predicted Changes in Aggregate Moments, $\gamma_f = 6.92$

	Early	Late	Late + γ_f
Inventor wage w	0.603	0.705	0.582
Prod. cutoff (exit)	1.004	1.257	1.020
Prod. cutoff (team)	1.579	1.710	1.325
Share of solo-based firms	93.7	84.7	83.6
Share of team patents	57.8	91.7	76.6
Employment share of firms with $n > 145$	32.8	79.2	44.1
Average firm size	7.1	25.1	9.7
Spillovers $z^{spillover}$	0.889	0.815	0.868
Output (normalized)	1.000	1.110	0.927

Notes: This table reports various aggregate moments of the competitive equilibrium. The first column is the early period calibration. The second column is the baseline late period calibration. The third column uses $\gamma_f = 6.92$ in the late period (targeting employment concentration) and changes the idea production parameters as in Table eftab:param.

B Supplementary Model Figures and Tables

B.1 Experiments with firm productivity distribution (γ_f)

This Appendix presents results from two additional experiments where γ_f, the shape of the Pareto distribution of firm productivities ε_f is allowed to vary between the early and late period. In the first experiment, we vary the idea production parameters as in Table 2 and then choose a new γ_f to exactly fit the level of employment concentration in the late period. This involves choose a thinner tail for the firm productivity distribution, lowering mean productivity of firms in the economy, because the idea production function changes overstate the rise in employment concentration, and less dispersed firm productivities in the late period are needed to lower employment concentration back down to the value observed in the data.

The results are in Table B2. Lower average firm productivity reduces inventors wages, contrary to the data, and output falls. The joint changes, though, get closer to matching the increase in firm size in the data (10.3 in the data), and the team patent share (77.6% in the data).

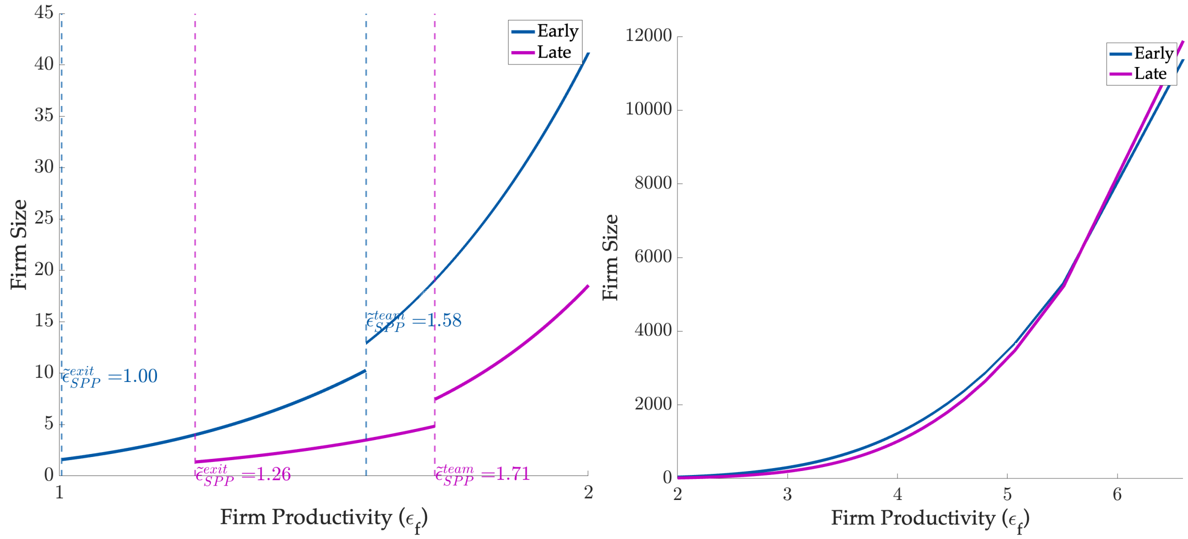
Table B3: Model Predicted Changes in Aggregate Moments, $\gamma_f = 5.77$

	Early	Late	$\gamma_f = 5.77$
Inventor wage w	0.603	0.705	0.638
Prod. cutoff (exit)	1.004	1.257	1.060
Prod. cutoff (team)	1.579	1.710	1.690
Share of solo-based firms	93.7	84.7	93.2
Share of team patents	57.8	91.7	66.3
Employment share of firms with $n > 145$	32.8	79.2	44.1
Average firm size	7.1	25.1	8.7
Spillovers $z^{spillover}$	0.889	0.815	0.878
Output (normalized)	1.000	1.110	1.052

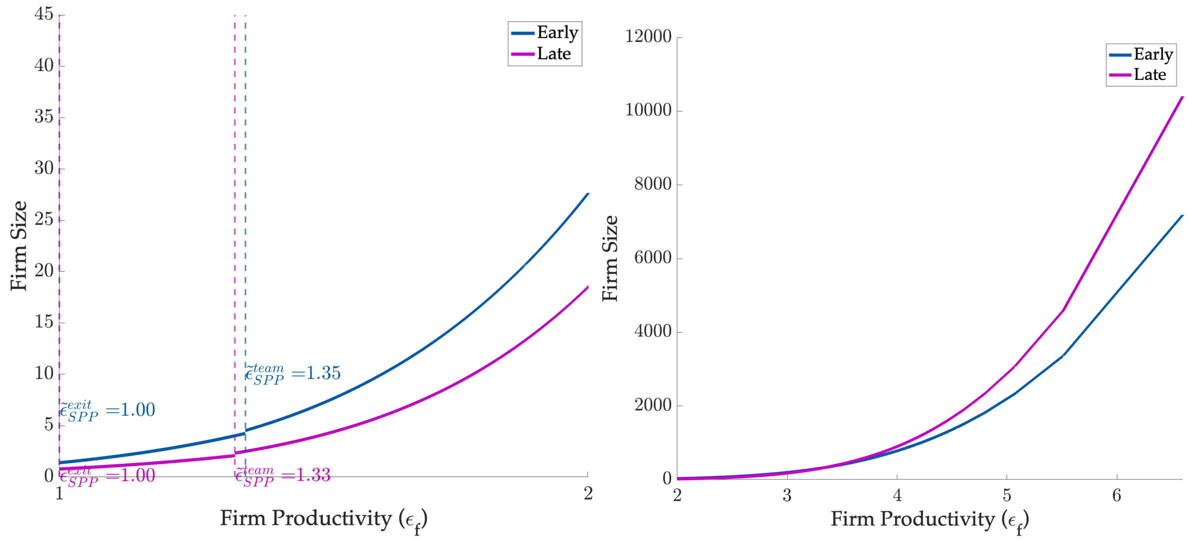
Notes: This table reports various aggregate moments of the competitive equilibrium. The first column is the early period calibration. The second column is the baseline late period calibration. The third column uses $\gamma_f = 5.77$ in the late period (targeting employment concentration) and holds the idea production parameters fixed to their early period values in Table eftab:param.

In a second experiment, we change γ_f by itself holding the idea production function parameters fixed at their early values to target employment concentration in the late period. In this case a lower value of $\gamma_f = 5.77$ is need to match the rise in concentration by making the firm productivity distribution more skewed (and raising average firm productivity). The results are in Table B3. The change in the firm productivity distribution to match higher concentration is consistent with a rise in the team patenting share and a modest rise in firm size, consistent with the data. However, the share of solo-based firms is almost unaffected, unlike in the data.

B.2 Size Decision Rules



(a) Competitive Equilibrium



(b) Social Optimum

Figure B4: Size Decision Rule: CE (top) vs. SPP (bottom) in Early and Late Periods

Note: These figures illustrate firm-size decision rules under the competitive equilibrium (top panel) and the social planner's allocation (bottom panel), for both the early and late periods. In each panel, the left subfigure focuses on the low-productivity (small-firm) region, while the right subfigure focuses on the high-productivity (large-firm) region to highlight the underlying patterns clearly. The blue lines correspond to the early period, and the purple lines indicate the late period. The vertical lines denote two productivity cutoffs for adopting team production and firm exit in each equilibrium for each period.

C Empirical Appendix

C.1 Computation of changes in minimum team productivity

The regression coefficient on "Team$_{ft}$ $\times$ Post$_t$" captures the change in log team productivity between the two periods, where team productivity $z_{team} = x_{min}^{team}\Gamma(1 - \frac{1}{\gamma^{team}})$. Call that coefficient R (in Table 1 column 1 $R = -0.049$). Then the distribution parameter in the late period $x_{min,L}^{team}$ is calculated as

$$
\begin{aligned}
R &= \log(x_{min,L}^{team}) + \log\left(\Gamma(1 - \frac{1}{\gamma_L^{team}})\right) - \log(x_{min,E}^{team}) - \log\left(\Gamma(1 - \frac{1}{\gamma_E^{team}})\right) \\
\log(x_{min,L}^{team}) &= R + \log(x_{min,E}^{team}) + \log(\Gamma(1 - \frac{1}{\gamma_E^{team}})) - \log(\Gamma(1 - \frac{1}{\gamma_L^{team}})) \\
x_{min,L}^{team} &= e^R x_{min,E}^{team} \frac{\Gamma(1 - \frac{1}{\gamma_E^{team}})}{\Gamma(1 - \frac{1}{\gamma_L^{team}})}.
\end{aligned}
$$

C.2 Robustness for Idea Production Function Estimation

Table C4: Returns to Scale in Team Production: Citations

	(1) All	(2) $n > 1$	(3) Public Firms
$\log(n_{ft})$	0.575***	0.675***	0.686***
	(0.003)	(0.004)	(0.006)
$\log(n_{ft}) \times \text{Team}_{ft}$	0.001	-0.070***	0.041***
	(0.004)	(0.005)	(0.007)
$\log(n_{ft}) \times \text{Team}_{ft} \times \text{Post}_t$	0.022***	0.001	0.003
	(0.005)	(0.006)	(0.008)
Team_{ft}	-0.429***	-0.236***	-0.472***
	(0.005)	(0.008)	(0.011)
Post_t	-0.095***	-0.188***	-0.105***
	(0.003)	(0.009)	(0.012)
$\log(n_{ft}) \times \text{Post}_t$	0.009***	0.026***	0.028***
	(0.003)	(0.004)	(0.005)
$\text{Team}_{ft} \times \text{Post}_t$	-0.042***	0.045***	-0.014
	(0.006)	(0.009)	(0.015)
$\log(K_{ft}^{intan.})$			-0.040***
			(0.003)
Constant	0.613***	0.379***	0.774***
	(0.003)	(0.008)	(0.021)
R-squared	0.452	0.485	0.616
Obs.	1105437	759085	131166
Firm FE	Yes	Yes	Yes

Notes: Column (1) includes all firm–year observations. Column (2) restricts the sample to firm–year observations with at least two inventors. Column (3) includes public firms with available intangible capital data from Peters and Taylor (2017). Outcome is 5-year forward citations purged of technology-class fixed effects. See equation (5.1) for details. Standard errors are reported in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

C.3 Team Diversity

Another trend over the past decades is rising knowledge diversity within teams. To measure knowledge diversity, we first identify the "core" CPC subgroup associated with an inventor in a given year as a proxy for the inventor's type or area of expertise.[16,17] The

[16] For example, G06N 20/00 is "Machine learning" or A23B 11/00 is "Preservation of milk or dairy products."

[17] For a patent associated with multiple CPC subgroups, we pick the main CPC subgroups based on the sequence variable provided by the USPTO.

Table C5: Returns to Scale in Team Production: Revelio Labs Data

	(1) All	(2) $n > 1$
$\log(n_{ft})$	0.353***	0.457***
	(0.010)	(0.012)
$\log(n_{ft}) \times$ Team$_{ft}$	0.169***	0.161***
	(0.007)	(0.009)
$\log(n_{ft}) \times$ Team$_{ft}$ $\times$ Post$_t$	0.041***	0.055***
	(0.010)	(0.014)
Team$_{ft}$	-0.071***	-0.096***
	(0.014)	(0.019)
Post$_t$	0.006	0.003
	(0.013)	(0.025)
$\log(n_{ft}) \times$ Post$_t$	0.019**	0.009
	(0.009)	(0.014)
Team$_{ft}$ $\times$ Post$_t$	0.023	-0.009
	(0.018)	(0.027)
Constant	0.026*	-0.254***
	(0.016)	(0.025)
R-squared	0.244	0.275
Obs.	169,800	135,406
Firm FE	Yes	Yes

Notes: Column (1) includes all firm–year observations. Column (2) restricts the sample to firm–year observations with at least two inventors. Outcome is number of patents. Data is Revelio Labs, 2008-2018, post-period is 2014-2018. See equation (5.1) for details. Standard errors are reported in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

core CPC subgroup of an inventor is defined as the subgroup with the highest number of patents filed by that inventor up to that year.[18] For each team, we calculate the number of core CPC subgroups associated with its inventors, which serves as a measure of the team's knowledge diversity in that year. We then average this knowledge diversity measure across all teams within a firm for that year. Finally, we compute the average of these firm-level measures across all firms in a given year.

[18]Eventually, we aim to leverage the Longitudinal Employer-Household Dynamics (LEHD) dataset, which contains detailed demographic and educational information for workers covered by the U.S. Unemployment Insurance (UI) system. By linking it to the Longitudinal Business Database (LBD) and USPTO inventor data, we plan to track workers' patenting activities within and across firms. This is in progress.

The result is plotted in Figure C5, which shows an upward trend and an increasing likelihood of the average team working with inventors with different specialties.

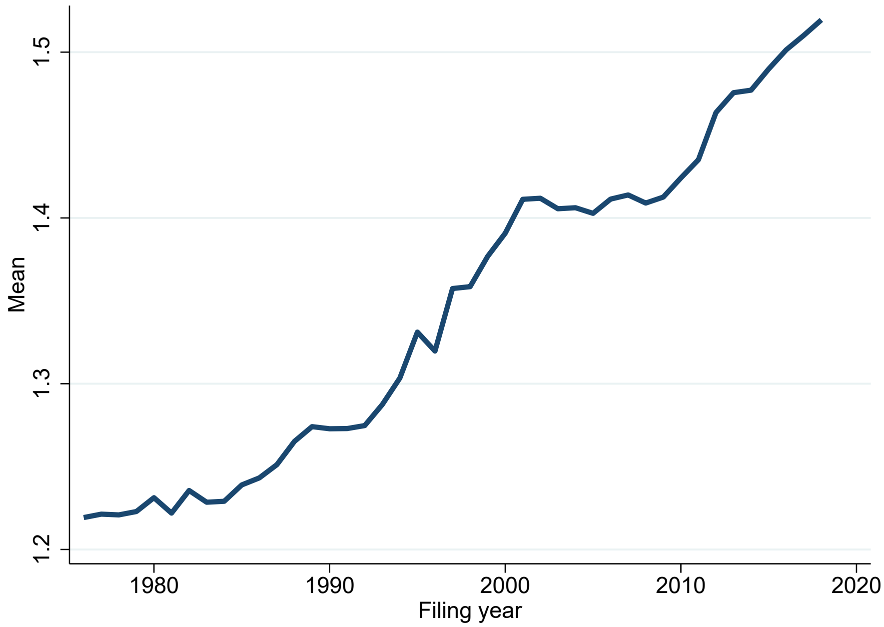


Figure C5: Average Number of Inventor CPC Subgroups in Teams in a Firm

Note: The figure illustrates the average number of core CPC subgroups associated with inventors on team patents in USPTO. A core CPC subgroup is defined as the modal CPC subgroup in an inventor's patenting history up to a given year. For each firm, we calculate the average number of core CPC subgroups for inventors on team patents and then compute the mean across all firms in a given year.