Online Appendix for
"Competition, Firm Innovation, and Growth under Imperfect Technology Spillovers"*

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A Baseline Model

A.1 Optimal Production and Employment

Final goods producer's production function is of the form:

$$Y = \frac{L^{\theta}}{1-\theta}\left[\int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j\in\mathcal{D}\}}\, dj \;+\; \int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j\notin\mathcal{D}\}}\, dj\right],$$

where $\mathcal{D}$ is the index set for differentiated products produced by domestic firms, and final good price is normalized to one $P = 1$. Thus profits are

$$\Pi^{\text{FG}} = Y = \frac{L^{\theta}}{1-\theta}\left[\int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j\in\mathcal{D}\}}\, dj \;+\; \int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j\notin\mathcal{D}\}}\, dj\right] - wL - \int_0^1 p_j\, y_j\, dj\,.$$

The FONCs of final good producer's profit maximization problem w.r.t. k_j and L are

$$\frac{\partial}{\partial y_j}: \quad p_j = q_j^{\theta}\, L^{\theta}\, y_j^{-\theta} \tag{A.1}$$

$$\frac{\partial}{\partial L}: \quad w = \frac{\theta}{1-\theta}\, L^{\theta-1}\left[\int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j\in\mathcal{D}\}}\, dj \;+\; \int_0^1 q_j^{\theta}\, y_j^{1-\theta}\, \mathcal{I}_{\{j\notin\mathcal{D}\}}\, dj\right]. \tag{A.2}$$

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Intermediate good producers, both domestic firms and foreign exporters, take differentiated product demand (A.1) as given and solve for the following profit maximization problem:

$$\pi(q_j) = \max_{y_j \geq 0} \left\{ L^\theta \, q_j^\theta \, y_j^{1-\theta} - y_j \right\} \, .$$

The FOC of this problem gives us:

$$\frac{\partial}{\partial y_j} : \quad (1-\theta) L^\theta \, q_j^\theta \, y_j^{-\theta} = 1 \quad \Rightarrow \quad y_j = (1-\theta)^{\frac{1}{\theta}} L \, q_j \text{ , and } p_j = \frac{1}{1-\theta} \, .$$

By plugging in the two optimal choices, differentiated product producer's profits from a product line j become

$$\pi(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}} \, L}_{\equiv \pi} \, q_j \, .$$

By plugging in optimal differentiated product production rule to (A.2), we get the wage rule that depends only on average product qualities

$$\begin{aligned} w &= \frac{\theta}{1-\theta} \, L^{\theta-1} \left[\int_0^1 q_j^\theta \, (1-\theta)^{\frac{1-\theta}{\theta}} \, L^{1-\theta} \, q_j^{1-\theta} \, \mathcal{I}_{\{j \in \mathcal{D}\}} \, dj + \int_0^1 q_j^\theta \, (1-\theta)^{\frac{1-\theta}{\theta}} \, L^{1-\theta} \, q_j^{1-\theta} \, \mathcal{I}_{\{j \notin \mathcal{D}\}} \, dj \right] \\ &= \frac{\theta}{1-\theta} \, L^{\theta-1} \, (1-\theta)^{\frac{1-\theta}{\theta}} \, L^{1-\theta} \, \int_0^1 q_j \, dj \\ &\Rightarrow w = \theta(1-\theta)^{1-2\theta} \overline{q} \end{aligned} \tag{A.3}$$

Finally, using the labor market clearing condition

$$L = 1 \, , \tag{A.4}$$

we get the equilibrium conditions:

$$Y = (1-\theta)^{\frac{1-2\theta}{\theta}} \overline{q} \tag{A.5}$$

$$y_j = (1-\theta)^{\frac{1}{\theta}} q_j \tag{A.6}$$

$$p_j = \frac{1}{1-\theta} \tag{A.7}$$

$$\pi = \theta(1-\theta)^{\frac{1-\theta}{\theta}} \, . \tag{A.8}$$

A.2 Product Quality Determination

This section lays out all possible cases in which a firm either retains or loses its product lines in the subsequent period. The probabilities are computed as functions of internal innovation intensities and the creative destruction arrival rate. Clearly, the past period technology gap $\Delta_t = \frac{q_t}{q_{t-1}}$ is the only information required to compute these probabilities. This is because in the competition between the incumbent firm and an outside firm attempting to take over the incumbent's product line, the next period product quality the incumbent and the outside firm can obtain is $q^{in}_{j,t+1} = \Delta_{j,t+1}\Delta_{j,t}q_{j,t-1}$ and $q^{en}_{j,t+1} = \eta q_{j,t-1}$, respectively.

A.2.1 Product Quality Evolution for Outsider Firms

Let's denote z^{ℓ}_j as the internal innovation intensity for product line j when it's technology gap is $\frac{q_{j,t}}{q_{j,t-1}} = \Delta^{\ell}$, such that $\Delta^1 = 1$, $\Delta^2 = \lambda$, $\Delta^3 = \eta$, and $\Delta^4 = \frac{\eta}{\lambda}$. Then, the evoluation of product quality in period $t+1$ occurs probabilistically as follows:

$$q_{j,t+1}(\Delta_t = 1) = \begin{cases} \lambda q_{j,t-1}\ , & \text{with prob. } (1-\overline{x})\, z^1_j \\ q_{j,t-1}\ , & \text{with prob. of } (1-\overline{x})\,(1-z^1_j) \\ \eta q_{j,t-1}\ , & \text{with prob. } \overline{x}\ , \end{cases}$$

where $q_{j,t-1} = q_{j,t}$,

$$q_{j,t+1}(\Delta_t = \lambda) = \begin{cases} \lambda^2 q_{j,t-1}\ , & \text{with prob. } z^2_j \\ \lambda q_{j,t-1}\ , & \text{with prob. } (1-\overline{x})\,(1-z^2_j) \\ \eta\, q_{j,t-1}\ , & \text{with prob. } \overline{x}(1-z^2_j)\ , \end{cases}$$

where $q_{j,t-1} = \frac{1}{\lambda} q_{j,t}$,

$$q_{j,t+1}(\Delta_t = 1+\eta) = \begin{cases} \lambda\eta\, q_{j,t-1}\ , & \text{with prob. } z^3_j \\ \eta\, q_{j,t-1}\ , & \text{with prob. } (1-\overline{x})(1-z^3_j) + \frac{1}{2}\,\overline{x}\,(1-z^3_j) \\ \eta\, q_{j,t-1}\ , & \text{with prob. } \frac{1}{2}\,\overline{x}\,(1-z^3_j)\ , \end{cases}$$

where $q_{j,t-1} = \frac{1}{\eta} q_{j.t}$, and

$$q_{j,t+1}\left(\Delta_t = \frac{\eta}{\lambda}\right) = \begin{cases} \lambda\, \frac{\eta}{\lambda}\, q_{j,t-1}\ , & \text{with prob. } (1-\overline{x})\, z^4_j + \frac{1}{2}\,\overline{x}\, z^4_j \\ \frac{\eta}{\lambda}\, q_{j,t-1}\ , & \text{with prob. } (1-\overline{x})\,(1-z^4_j) \\ \eta q_{j,t-1}\ , & \text{with prob. of } \overline{x}\,(1-z^4_j) + \frac{1}{2}\,\overline{x}\, z^4_j\ , \end{cases}$$

where $q_{j,t-1} = \frac{\lambda}{1+\eta} q_{j,t}$.

A.2.2 Product Quality Evolution for an Incumbent Firm

For each Δ^{ℓ}, the transition dynamics for product quality and technology gap for product line j_i can be represented using two indicator functions I^z_i and $I^{\overline{x}}_i$, where $\Delta'_{j_i} = 0$ (or equivalently $\{q'_{j_i}\} = \phi$) implies firm loses product line j_i in the next period. The expressions are formulated as if the incumbent firm is engaging in coin-tossing at all times.

i) $\Delta_{j_i} = \Delta^1 = 1$

$I_i^{\overline{x}}$	I_i^z	prob. $\frac{1}{2}$ (win)	prob. $\frac{1}{2}$ (lose)
1	0	$\Delta'_{j_i} = 0$	$\Delta'_{j_i} = 0$
1	1	$\Delta'_{j_i} = 0$	$\Delta'_{j_i} = 0$
0	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 1$
0	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$

$$\Rightarrow \begin{cases} \Delta'_{j_i} = (1 - I_j^{\overline{x}})(\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (win)} \\ \Delta'_{j_i} = (1 - I_j^{\overline{x}})(\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (lose)} \\ \{q'_{j_i}\} = \left\{(1 - I_j^{\overline{x}})(\lambda I_i^z)\, q_{j_i}\right\} \setminus \{0\} & \text{prob. } \frac{1}{2} \text{ (win)} \\ \{q'_{j_i}\} = \left\{(1 - I_j^{\overline{x}})(\lambda I_i^z)\, q_{j_i}\right\} \setminus \{0\} & \text{prob. } \frac{1}{2} \text{ (lose)} \end{cases}$$

ii) $\Delta_{j_i} = \Delta^2 = \lambda$

$I_i^{\overline{x}}$	I_i^z	prob. $\frac{1}{2}$ (win)	prob. $\frac{1}{2}$ (lose)
1	0	$\Delta'_{j_i} = 0$	$\Delta'_{j_i} = 0$
1	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$
0	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 1$
0	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$

$$\Rightarrow \begin{cases} \Delta'_{j_i} = \left[1 - (1 - I_i^z) I_j^{\overline{x}}\right](\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (win)} \\ \Delta'_{j_i} = \left[1 - (1 - I_i^z) I_j^{\overline{x}}\right](\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (lose)} \\ \{q'_{j_i}\} = \left\{\left[1 - (1 - I_i^z) I_j^{\overline{x}}\right](\lambda I_i^z)\, q_{j_i}\right\} \setminus \{0\} & \text{prob. } \frac{1}{2} \text{ (win)} \\ \{q'_{j_i}\} = \left\{\left[1 - (1 - I_i^z) I_j^{\overline{x}}\right](\lambda I_i^z)\, q_{j_i}\right\} \setminus \{0\} & \text{prob. } \frac{1}{2} \text{ (lose)} \end{cases}$$

iii) $\Delta_{j_i} = \Delta^3 = \eta$

$I_i^{\overline{x}}$	I_i^z	prob. $\frac{1}{2}$ (win)	prob. $\frac{1}{2}$ (lose)
1	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 0$
1	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$
0	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 1$
0	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$

$$
\Rightarrow \begin{cases} \Delta'_{j_i} = \lambda I_i^z & \text{prob. } \frac{1}{2} \text{ (win)} \\ \Delta'_{j_i} = \left[1-(1-I_i^z)I_j^{\overline{x}}\right](\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (lose)} \\ \{q'_{j_i}\} = \left\{(\lambda I_i^z)\, q_{j_i}\right\} \setminus \{0\} & \text{prob. } \frac{1}{2} \text{ (win)} \\ \{q'_{j_i}\} = \left\{\left[1-(1-I_i^z)I_j^{\overline{x}}\right](\lambda I_i^z)\, q_{j_i}\right\} \setminus \{0\} & \text{prob. } \frac{1}{2} \text{ (lose)} \end{cases}
$$

iv) $\Delta_{j_i} = \Delta^4 = \frac{\eta}{\lambda}$

$I_i^{\overline{x}}$	I_i^z	prob. $\frac{1}{2}$ (win)	prob. $\frac{1}{2}$ (lose)
1	0	$\Delta'_{j_i} = 0$	$\Delta'_{j_i} = 0$
1	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = 0$
0	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 1$
0	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$

$$
\Rightarrow \begin{cases} \Delta'_{j_i} = \left[1-(1-I_i^z)I_j^{\overline{x}}\right](\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (win)} \\ \Delta'_{j_i} = (1-I_i^{\overline{x}})(\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (lose)} \\ \{q'_{j_i}\} = \left\{\left[1-(1-I_i^z)I_j^{\overline{x}}\right](\lambda I_i^z)\, q_{j_i}\right\} \setminus \{0\} & \text{prob. } \frac{1}{2} \text{ (win)} \\ \{q'_{j_i}\} = \left\{(1-I_i^{\overline{x}})(\lambda I_i^z)\, q_{j_i}\right\} \setminus \{0\} & \text{prob. } \frac{1}{2} \text{ (lose)} \end{cases}
$$

A.3 Value Function and Optimal Innovation Decisions

The conditional expectation inside the expression for the value function considers the success/failure of internal and external innovation, the arrival of the creative destruction shock, outcomes of coin-tosses (c-t), the distribution of current period product quality q, and the distribution of the current period technology gap Δ^ℓ. Thus $\mathbb{E}\left[V\left(\Phi^{f\prime} \mid \Phi^f\right) \Big| \{z_j\}_{j\in\mathcal{J}^f}, x\right]$ is equivalent to

$$
\sum_{I_1^{\overline{x}}, I_2^{\overline{x}}, \ldots, I_{n_f}^{\overline{x}}=0}^{1} \; \sum_{I_1^z, \ldots, I_{n_f}^z=0}^{1} \; \sum_{\text{c-t}_1, \ldots, \text{c-t}_{n_f} = \text{win}}^{\text{lose}} \; \sum_{I^x=0}^{1} \left[\prod_{i=1}^{n_f} \overline{x}^{I_i^{\overline{x}}} (1-\overline{x})^{1-I_i^{\overline{x}}} z_i^{I_i^z} (1-z_i)^{1-I_i^z}\right] \times \left[x^{I^x}(1-x)^{1-I^x}\right] \left(\frac{1}{2}\right)^{n_f}
$$

$$
\times\, \mathbb{E}_{q,\Delta}\, V\left(\left[\bigcup_{i=1}^{n_f}\left[\left\{\left(\Delta'_{j_i} q_{j_i}, \Delta'_{j_i}\right) \mid (q_{j_i}, \Delta_{j_i}), I_i^{\overline{x}}, I_i^z, \text{c-t}_i\right\} \setminus \{\mathbf{0}\}\right]\right] \bigcup \left[\left\{\left(\frac{\eta}{\Delta_{-j}}\, I^x\, q_{-j},\; \frac{\eta}{\Delta_{-j}} I^x\right)\right\} \setminus \{\mathbf{0}\}\right]\right).
$$

The first term inside of the value function, $\bigcup_{i=1}^{n_f}\left[\left\{\left(\Delta'_{j_i} q_{j_i}, \Delta'_{j_i}\right) \mid (q_{j_i}, \Delta_{j_i}), I_i^{\overline{x}}, I_i^z, \text{c-t}_i\right\} \setminus \{\mathbf{0}\}\right]$, depicts the subsets of possible realizations for $\Phi^{f\prime}$ from internal innovation, creative destruction, and coin-toss. The second term, $\left\{\left(\frac{\eta}{\Delta_{-j}}\, I^x\, q_{-j},\; \frac{\eta}{\Delta_{-j}} I^x\right)\right\} \setminus \{\mathbf{0}\}$, shows the subsets of possible realizations for $\Phi^{f\prime}$ from external innovation, where $\{q'_{j_i}\} = \{\Delta'_{j_i} q_{j_i}\} \setminus \{0\}$, and $\{q'_{-j}\} = \{\frac{\eta}{\Delta_{-j}}\, I^x\, q_{-j}\} \setminus \{0\}$. If $\Delta'_{j_i} = 0$, then firm f loses product line j_i and $\{(q'_{j_i}, \Delta'_{j_i})\} \setminus \{\mathbf{0}\} = \{\mathbf{0}\} \setminus \{\mathbf{0}\} = \varnothing$.

A.4 Potential Startups

By plugging in the value function defined in the previous section, the expected term becomes

$$\begin{aligned}\mathbb{E}V(\{(q_j', \Delta_j')\}) = \mathbb{E}_{q_j}\Bigg[&\frac{1}{2}x_e(1-z^3)\Big[A_1 q_j + B\overline{q}'\Big]\mu(\Delta^3) + x_e\left(1-\frac{1}{2}z^4\right)\Big[A_2\lambda g_j + B\overline{q}'\Big]\mu(\Delta^4)\\
&+ x_e\Big[A_3\eta q_j + B\overline{q}'\Big]\mu(\Delta^1) + x_e(1-z^2)\Big[A_4\frac{\eta}{\lambda}q_j + B\overline{q}'\Big]\mu(\Delta^2)\Bigg]\\
= x_e&\left[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right]\overline{q}\\
+ x_e&\left[\frac{1}{2}(1-z^3)\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)\mu(\Delta^4) + \mu(\Delta^1) + (1-z^2)\mu(\Delta^2)\right]B(1+g)\overline{q}\,.\end{aligned}$$

Thus from the FOSC, the optimal external innovation intensity for potential startups x_e is

$$\begin{aligned}x_e = \Bigg[\Bigg[&\left(\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right)\\
&+\left(\frac{1}{2}(1-z^3)\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)\mu(\Delta^4) + \mu(\Delta^1) + (1-z^2)\mu(\Delta^2)\right)B(1+g)\Bigg] \times \frac{\widetilde{\beta}}{\widetilde{\psi}_e\,\widetilde{\chi}_e}\Bigg]^{\frac{1}{\widetilde{\psi}_e - 1}}.\end{aligned}$$

A.5 Technology Gap Portfolio Composition Distribution Transition

Note that the range of $\widetilde{k}^1$ follows the fact that

i. For $0 \le \widetilde{k} \le \min\{n_f - k, k\}$, the two combinations preceding the term in brackets are well defined for any $\widetilde{k}^1 \in [0, \widetilde{k}] \cap \mathbb{Z}$ and describe all the possible cases.

ii. If $n_f - k \ge k$, then $\widetilde{k} > k$, $0 \le \widetilde{k} - \widetilde{k}^1$, and $0 \le \widetilde{k}^1 \le n_f - k$ should be satisfied. Thus $\widetilde{k} - k \le \widetilde{k}^1 \le \widetilde{k}$.

iii. If $k \ge n_f - k$, then $\widetilde{k} > n_f - k$, $0 \le \widetilde{k} - \widetilde{k}^1$, and $0 \le \widetilde{k}^1 \le n_f - k$ should be satisfied. Thus $\max\{0,\, \widetilde{k} - k\} \le \widetilde{k}^1 \le n_f - k$.

Now, let's define technology gap portfolio composition with $n_f - k$ numbers of $\Delta = \Delta^1$, k numbers of $\Delta = \Delta^2$, zero number of $\Delta = \Delta^3$ and $\Delta = \Delta^4$ as $\widetilde{\mathcal{N}}(n_f, k) \equiv (n_f, n_f - k, k, 0, 0)$, where $k \in [0, n_f] \cap \mathbb{Z}$, $n_f > 0$. Then without considering external innovation, the probability of $\mathcal{N} = \widetilde{\mathcal{N}}(n_f, k)$ transitioning to $\mathcal{N}' = \widetilde{\mathcal{N}}(n_f, \widetilde{k})$ can be computed as:

$$\widetilde{\mathbb{P}}\Big(n_f, \widetilde{k} \mid n_f, k\Big) =$$

$$
\begin{cases}
\sum_{\widetilde{k}^1=\max\{0,\,\widetilde{k}-k\}}^{\min\{n_f-k,\,\widetilde{k}\}} \begin{pmatrix} n_f-k \\ \widetilde{k}^1 \end{pmatrix} \begin{pmatrix} k \\ \widetilde{k}-\widetilde{k}^1 \end{pmatrix} \\
\quad \times \left[\begin{array}{r} (1-\overline{x})^{n_f-(\widetilde{k}-\widetilde{k}^1)}\,(1-z^1)^{n_f-k-\widetilde{k}^1}\,(z^1)^{\widetilde{k}^1} \\ \times(1-z^2)^{k-(\widetilde{k}-\widetilde{k}^1)}\,(z^2)^{\widetilde{k}-\widetilde{k}^1} \end{array} \right] & \text{for } n_f \geq 1, \text{ and } 0 \leq \widetilde{k}, k \leq n_f \\
0 & \text{otherwise,}
\end{cases}
$$

where

$$
\begin{pmatrix} n \\ k \end{pmatrix} = \frac{n!}{k!(n-k)!}
$$

is a combination of selecting k elements from n elements without repetition as the order of selection does not matter. The range for $\widetilde{k}^1$ follows the form described as above due to the fact that:

i. For $0 \leq \widetilde{k} \leq \min\{n_f-k, k\}$ case, the two combinations are well defined for any $\widetilde{k}^1 \in [0, \widetilde{k}] \cap \mathbb{Z}$ and encompass all the possible cases.

ii. For $n_f - k \geq k$ case, $\tilde{k} > k$, $0 \leq \widetilde{k} - \widetilde{k}^1$, and $0 \leq \widetilde{k}^1 \leq n_f - k$ should be satisfied. Thus $\widetilde{k} - k \leq \widetilde{k}^1 \leq \widetilde{k}$.

iii. For $k \geq n_f - k$ case, $\widetilde{k} > n_f - k$, $0 \leq \widetilde{k} - \widetilde{k}^1$, and $0 \leq \widetilde{k}^1 \leq n_f - k$ should be satisfied. Thus $\max\{0,\, \widetilde{k} - k\} \leq \widetilde{k}^1 \leq n_f - k$.

By using $\widetilde{\mathbb{P}}(n_f, \widetilde{k} \mid n_f, k)$, the probability of $\mathcal{N} = \widetilde{\mathcal{N}}(n_f,\ k)$ transitioning to $\mathcal{N}' = \widetilde{\mathcal{N}}(n_f - h,\ \widetilde{k})$ for any $h \geq 0$ without considering external innovation can be defined as follows. Take out h^1 numbers of product lines with $\Delta = \Delta^1$, and $h - h^1$ numbers of product lines with $\Delta = \Delta^2$ from $\widetilde{\mathcal{N}}(n_f,\ k)$, then compute the probability of $\widetilde{\mathcal{N}}(n_f - h,\ k - (h - h^1))$ transitioning to $\widetilde{\mathcal{N}}(n_f - h,\ \widetilde{k})$ with $\widetilde{\mathbb{P}}\Big(n_f - h,\ \widetilde{k} \mid n_f - h,\ k - (h - h^1)\Big)$ for all feasible h^1:

$$
\widetilde{\mathbb{P}}\Big(n_f - h,\ \widetilde{k} \mid n_f,\ k\Big) =
$$

$$
\begin{cases}
\sum_{h^1=\max\{0,\,h-k\}}^{\min\{h,\,n_f-k\}} \Bigg[\begin{pmatrix} n_f-k \\ h^1 \end{pmatrix} \begin{pmatrix} k \\ h-h^1 \end{pmatrix} \overline{x}^h (1-z^2)^{h-h^1} \\
\qquad \times \widetilde{\mathbb{P}}\Big(n_f-h,\ \widetilde{k} \mid n_f-h,\ k-(h-h^1)\Big) \Bigg] & \text{for } 0 \leq h < n_f, \\
& n_f \geq 1, \\
& 0 \leq \widetilde{k} \leq n_f - h, \\
& \text{and } 0 \leq k \leq n_f \\
\overline{x}^{n_f}\,(1-z^2)^k & \text{for } h = n_f \geq 1, \\
& \widetilde{k} = 0, \\
& \text{and } 0 \leq k \leq n_f \\
0 & \text{otherwise.}
\end{cases}
$$

The range for h^1 is defined as above, following that for any h^1, $0 \leq h - h^1 \leq k$ and $0 \leq h^1 \leq n_f - k$ should be satisfied.

Using $\widetilde{\mathbb{P}}\Big(n_f - h,\ \widetilde{k} \mid n_f,\ k\Big)$, other possible technology gap portfolio composition transition probabilities can be described in a convenient manner as follows:

1-i. The probability of $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ to $\mathcal{N}' = (n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0)$ for $h \geq -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0 \ \Big|\, n_f, n_f - k, k, 0, 0\Big) =$$

$$\begin{aligned} &\widetilde{\mathbb{P}}\Big(n_f - h, \widetilde{k} \,\Big|\, n_f, k\Big)\, (1 - x\overline{x}_{takeover}) \\ &+ \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f, k\Big)\, \mu(\Delta^3)\, \frac{1}{2}\, x\, (1 - z^3) \\ &+ \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} - 1 \,\Big|\, n_f, k\Big)\, \mu(\Delta^4)\, x\, \left(1 - \frac{1}{2} z^4\right) . \end{aligned}$$

The first term is the probability of $\mathcal{N}$ transitioning to $\mathcal{N}'$ directly via the change in the firm's existing technology gap portfolio composition, but with unsuccessful external innovation. The second term is the probability of $\mathcal{N}$ to $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k})$, where successful external innovation adds one product line with $\Delta' = \Delta^1$. Since the next period technology gap of product line j from successful external innovation is equal to $\Delta_j' = \frac{q_j'}{q_j} = \frac{\eta q_{j,-1}}{\Delta_j q_{j,-1}} = \frac{\eta}{\Delta_j}$, firm needs to take over a product line with a technology gap of $\Delta = \Delta^3 = 1 + \eta$ to have a product line with a technology gap of Δ^1 in the next period. The third term is the probability of $\mathcal{N}$ to $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k} - 1)$, then successful external innovation adds one product line with $\Delta' = \Delta^2$ by taking over a product line with a technology gap of $\Delta = \Delta^4$. For $h = -1$, the first term becomes zero by the definition of $\widetilde{\mathbb{P}}(\,\cdot \mid \cdot\,)$. Thus this probability is well defined for any $h \geq -1$.

1-ii. The probability of $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0)$ for $h \geq -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0 \,\Big|\, n_f, n_f - k, k, 0, 0\Big) = \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f, k\Big)\, \mu(\Delta^1)\, x\, .$$

The firm's existing technology gap changes from $\widetilde{\mathcal{N}}(n_f, k)$ to $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k})$, then successful external innovation adds $\Delta' = \Delta^3 = 1 + \eta$.

1-iii. The probability of $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ to $\mathcal{N}' = (n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 0, 1)$ for $h \geq -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 0, 1 \,\Big|\, n_f, n_f - k, k, 0, 0\Big) =$$

$$\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f, k\Big)\, \mu(\Delta^2)\, x\, (1 - z^2)\, .$$

2-i. For $n_f \geq 2$, the probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 1, 0)$ to $\mathcal{N}' = (n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0)$ for $h \geq -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0 \,\Big|\, n_f, n_f - 1 - k, k, 1, 0\Big) =$$

$$
\begin{bmatrix}
\widetilde{\mathbb{P}}\Big(n_f - h, \widetilde{k} \,\big|\, n_f - 1, k\Big)\, \tfrac{1}{2}\,\overline{x}\,(1 - z^3) \\
+\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\big|\, n_f - 1, k\Big)\, \left(1 - \tfrac{1}{2}\overline{x}\right)\, (1 - z^3) \\
+\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} - 1 \,\big|\, n_f - 1, k\Big)\, z^3
\end{bmatrix} \times (1 - x\,\overline{x}_{takeover})
$$

$$
+ \begin{bmatrix}
\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\big|\, n_f - 1, k\Big)\, \tfrac{1}{2}\,\overline{x}\,(1 - z^3) \\
+\widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} \,\big|\, n_f - 1, k\Big)\, \left(1 - \tfrac{1}{2}\overline{x}\right)\, (1 - z^3) \\
+\widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\big|\, n_f - 1, k\Big)\, z^3
\end{bmatrix} \times \mu(\Delta^3)\, \frac{1}{2}\, x\,(1 - z^3)
$$

$$
+ \begin{bmatrix}
\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} - 1 \,\big|\, n_f - 1, k\Big)\, \tfrac{1}{2}\,\overline{x}\,(1 - z^3) \\
+\widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\big|\, n_f - 1, k\Big)\, \left(1 - \tfrac{1}{2}\overline{x}\right)\, (1 - z^3) \\
+\widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 2 \,\big|\, n_f - 1, k\Big)\, z^3
\end{bmatrix} \times \mu(\Delta^4)\, x\, \left(1 - \frac{1}{2} z^4\right) .
$$

The three probabilities within the brackets represent the probabilities when the existing product line with $\Delta = \Delta^3$ is taken over by other firm, internal innovation fails but the firm retains it, and internal innovation succeeds and the firm retains it. The first bracket is the probability of $\mathcal{N}$ transitioning to $\mathcal{N}'$ when external innovation fails. The second bracket is the probability of $\mathcal{N}$ transitioning to $\mathcal{N}'$ when successful external innovation adds a product line with a technology gap of $\Delta' = \Delta^1$, and the third bracket is the probability of $\mathcal{N}$ transitioning to $\mathcal{N}'$ when successful external innovation adds a product line with $\Delta' = \Delta^2$. Similarly, for $n_f = 1$,

$$
\begin{aligned}
\mathbb{P}\Big(1, 1, 0, 0, 0 \,\big|\, 1, 0, 0, 1, 0\Big) = &\left(1 - \frac{1}{2}\overline{x}\right) (1 - z^3)(1 - x\,\overline{x}_{takeover}) \\
&+ \frac{1}{2}\,\overline{x}\,(1 - z^3)\,\mu(\Delta^3)\,\frac{1}{2}\,x\,(1 - z^3)\,,
\end{aligned}
$$

and

$$
\mathbb{P}\Big(1, 0, 1, 0, 0 \,\big|\, 1, 0, 0, 1, 0\Big) = z^3\,(1 - x\,\overline{x}_{takeover}) + \frac{1}{2}\,\overline{x}\,(1 - z^3)\,\mu(\Delta^4)\,x\,\left(1 - \frac{1}{2} z^4\right) .
$$

2-ii. For $n_f \geq 2$, the probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 1, 0)$ to $\mathcal{N}' = (n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0)$ for $h \geq -1$ is defined as

$$
\begin{aligned}
&\mathbb{P}\Big(n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0 \,\big|\, n_f, n_f - 1 - k, k, 1, 0\Big) = \\
&\qquad \begin{bmatrix}
\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\big|\, n_f - 1, k\Big)\, \tfrac{1}{2}\,\overline{x}\,(1 - z^3) \\
+\widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} \,\big|\, n_f - 1, k\Big)\, \left(1 - \tfrac{1}{2}\overline{x}\right)\, (1 - z^3) \\
+\widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\big|\, n_f - 1, k\Big)\, z^3
\end{bmatrix} \times \mu(\Delta^1)\, x.
\end{aligned}
$$

If $\mathcal{N}$ transitions to $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k})$ through internal innovation, then successful external innovation adds a product line with $\Delta' = \Delta^3$ by taking over a product line with $\Delta = \Delta^1$. Similarly, for $n_f = 1$,

$$
\mathbb{P}\Big(1, 0, 0, 1, 0 \,\big|\, 1, 0, 0, 1, 0\Big) = \frac{1}{2}\,\overline{x}\,(1 - z^3)\,\mu(\Delta^1)\,x\,.
$$

2-iii. For $n_f \geq 2$, the probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 1, 0)$ to $\mathcal{N}' = (n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 0, 1)$ for $h \geq -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 0, 1 \,\Big|\, n_f, n_f - 1 - k, k, 1, 0\Big) =$$
$$\left[\begin{array}{l} \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f - 1, k\Big) \, \tfrac{1}{2}\, \overline{x}\, (1 - z^3) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} \,\Big|\, n_f - 1, k\Big) \, \left(1 - \tfrac{1}{2}\overline{x}\right) \, (1 - z^3) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big) \, z^3 \end{array}\right] \times \mu(\Delta^2)\, x\, (1 - z^2)\,.$$

Similarly, for $n_f = 1$,

$$\mathbb{P}\Big(1, 0, 0, 0, 1 \,\Big|\, 1, 0, 0, 1, 0\Big) = \frac{1}{2}\, \overline{x}\, (1 - z^3)\, \mu(\Delta^2)\, x\, (1 - z^2)\,.$$

3-i. For $n_f \geq 2$, the probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 0, 1)$ to $\mathcal{N}' = (n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0)$ for $h \geq -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0 \,\Big|\, n_f, n_f - 1 - k, k, 0, 1\Big) =$$
$$\left[\begin{array}{l} \widetilde{\mathbb{P}}\Big(n_f - h, \widetilde{k} \,\Big|\, n_f - 1, k\Big) \, \overline{x} \, \left(1 - \tfrac{1}{2} z^4\right) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f - 1, k\Big) \, (1 - \overline{x})\, (1 - z^4) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big) \, \left(1 - \tfrac{1}{2}\overline{x}\right) \, z^4 \end{array}\right] \times (1 - x\, \overline{x}_{takeover})$$
$$+ \left[\begin{array}{l} \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f - 1, k\Big) \, \overline{x} \, \left(1 - \tfrac{1}{2} z^4\right) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} \,\Big|\, n_f - 1, k\Big) \, (1 - \overline{x})\, (1 - z^4) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big) \, \left(1 - \tfrac{1}{2}\overline{x}\right) \, z^4 \end{array}\right] \times \mu(\Delta^3)\, \frac{1}{2}\, x\, (1 - z^3)$$
$$+ \left[\begin{array}{l} \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big) \, \overline{x} \, \left(1 - \tfrac{1}{2} z^4\right) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big) \, (1 - \overline{x})\, (1 - z^4) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 2 \,\Big|\, n_f - 1, k\Big) \, \left(1 - \tfrac{1}{2}\overline{x}\right) \, z^4 \end{array}\right] \times \mu(\Delta^4)\, x\, \left(1 - \frac{1}{2} z^4\right)\,.$$

Similarly, for $n_f = 1$,

$$\mathbb{P}\Big(1, 1, 0, 0, 0 \,\Big|\, 1, 0, 0, 0, 1\Big) = (1 - \overline{x})(1 - z^4)(1 - x\, \overline{x}_{takeover}) + \overline{x}\left(1 - \frac{1}{2} z^4\right) \mu(\Delta^3) \frac{1}{2}\, x\, (1 - z^3)$$

and

$$\mathbb{P}\Big(1, 0, 1, 0, 0 \,\Big|\, 1, 0, 0, 0, 1\Big) = \left(1 - \frac{1}{2}\overline{x}\right) z^4 (1 - x\, \overline{x}_{takeover}) + \overline{x}\left(1 - \frac{1}{2} z^4\right) \mu(\Delta^4)\, x\, \left(1 - \frac{1}{2} z^4\right)\,.$$

3-ii. For $n_f \geq 2$, the probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 0, 1)$ to $\mathcal{N}' = (n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0)$ for $h \geq -1$

is defined as

$$\mathbb{P}\Big(n_f-h,n_f-h-1-\widetilde{k},\widetilde{k},1,0 \,\big|\, n_f,n_f-1-k,k,0,1\Big) = \begin{bmatrix} \widetilde{\mathbb{P}}\Big(n_f-h-1,\widetilde{k} \,\big|\, n_f-1,k\Big)\,\overline{x}\,\left(1-\frac{1}{2}z^4\right) \\ +\widetilde{\mathbb{P}}\Big(n_f-h-2,\widetilde{k} \,\big|\, n_f-1,k\Big)\,(1-\overline{x})\,(1-z^4) \\ +\widetilde{\mathbb{P}}\Big(n_f-h-2,\widetilde{k}-1 \,\big|\, n_f-1,k\Big)\,\left(1-\frac{1}{2}\overline{x}\right)\,z^4 \end{bmatrix} \times \mu(\Delta^1)\,x\,.$$

Similarly, for $n_f=1$,

$$\mathbb{P}\Big(1,0,0,1,0 \,\big|\, 1,0,0,0,1\Big) = \overline{x}\left(1-\frac{1}{2}z^4\right)\mu(\Delta^1)\,x\,.$$

3-iii. For $n_f \geq 2$, the probability of $\mathcal{N}=(n_f,n_f-1-k,k,0,1)$ to $\mathcal{N}'=(n_f-h,n_f-h-1-\widetilde{k},\widetilde{k},0,1)$ for $h\geq -1$ is defined as

$$\mathbb{P}\Big(n_f-h,n_f-h-1-\widetilde{k},\widetilde{k},0,1 \,\big|\, n_f,n_f-1-k,k,0,1\Big) = \begin{bmatrix} \widetilde{\mathbb{P}}\Big(n_f-h-1,\widetilde{k} \,\big|\, n_f-1,k\Big)\,\overline{x}\,\left(1-\frac{1}{2}z^4\right) \\ +\widetilde{\mathbb{P}}\Big(n_f-h-2,\widetilde{k} \,\big|\, n_f-1,k\Big)\,(1-\overline{x})\,(1-z^4) \\ +\widetilde{\mathbb{P}}\Big(n_f-h-2,\widetilde{k}-1 \,\big|\, n_f-1,k\Big)\,\left(1-\frac{1}{2}\overline{x}\right)\,z^4 \end{bmatrix} \times \mu(\Delta^2)\,x\,(1-z^2)\,.$$

Similarly, for $n_f=1$,

$$\mathbb{P}\Big(1,0,0,0,1 \,\big|\, 1,0,0,0,1\Big) = \overline{x}\left(1-\frac{1}{2}z^4\right)\mu(\Delta^2)\,x\,(1-z^2)\,.$$

Now that the probabilities of any specific technology gap portfolio composition transitioning to another specific technology gap portfolio composition are computed, we can specify the inflows and outflows of a particular technology gap portfolio. Let $\mathcal{F}$ be the total mass of firms in the economy, and let $\mu(\mathcal{N})$ be the share of firms with technology gap portfolio $\mathcal{N}$. Thus, $\widetilde{\mu}(\mathcal{N})=\mathcal{F}\mu(\mathcal{N})$.

i) For $\mathcal{N}=(n_f,n_f-k,k,0,0)$ with $n_f\geq 2$, any firms with a technology gap portfolio next period not equal to $\mathcal{N}$ accounts for outflows. Thus,

$$\begin{aligned}\text{outflow}\big(n_f,n_f-k,k,0,0\big) =& \Big[\,1-\mathbb{P}\big(n_f,n_f-k,k,0,0 \,\big|\, n_f,n_f-k,k,0,0\big)\Big] \\ & \times \mathcal{F}\,\mu\big(n_f,n_f-k,k,0,0\big)\,.\end{aligned}$$

Any firms with a total number of product line $n\geq n_f-1$ can have a technology gap portfolio composition equal to $\mathcal{N}$ through combinations of internal and external innovations. Thus, for the maximum number of product lines $\overline{n}_f$:

$$\text{inflow}\big(n_f,n_f-k,k,0,0\big) = \mathcal{F}\sum_{n=n_f-1}^{\overline{n}_f}\sum_{\widetilde{k}=0}^{n}\Big[\,\mu\big(n,n-\widetilde{k},\widetilde{k},0,0\big)\,\mathbb{P}\big(n_f,n_f-k,k,0,0 \,\big|\, n,n-\widetilde{k},\widetilde{k},0,0\big)$$

$$
\begin{aligned}
&+ \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&\times \mathbb{P}\big(n_f, n_f - k, k, 0, 0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&+ \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \\
&\times \mathbb{P}\big(n_f, n_f - k, k, 0, 0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \Bigg] \\
&- \mathcal{F}\, \mu\big(n_f, n_f - k, k, 0, 0\big)\, \mathbb{P}\big(n_f, n_f - k, k, 0, 0 \,\big|\, n_f, n_f - k, k, 0, 0\big)\,.
\end{aligned}
$$

ii) $\mathcal{N} = (n_f, n_f - 1 - k, k, 1, 0)$ with $n_f \geq 2$

$$
\begin{aligned}
\text{outflow}\big(n_f, n_f - 1 - k, k, 1, 0\big) = &\Big[\, 1 - \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n_f, n_f - 1 - k, k, 1, 0\big) \Big] \\
&\times \mathcal{F}\, \mu\big(n_f, n_f - 1 - k, k, 1, 0\big)\,.
\end{aligned}
$$

$$
\begin{aligned}
\text{inflow}\big(n_f, n_f - 1 - k, k, 1, 0\big) = \mathcal{F} \sum_{n=n_f-1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} &\Bigg[\, \mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big) \\
&+ \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&\times \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&+ \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \\
&\times \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \Bigg] \\
&- \mathcal{F}\, \mu\big(n_f, n_f - 1 - k, k, 1, 0\big)\, \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n_f, n_f - 1 - k, k, 1, 0\big)\,.
\end{aligned}
$$

iii) $\mathcal{N} = (n_f, n_f - 1 - k, k, 0, 1)$ with $n_f \geq 2$

$$
\begin{aligned}
\text{outflow}\big(n_f, n_f - 1 - k, k, 0, 1\big) = &\Big[\, 1 - \mathbb{P}\big(n_f, n_f - 1 - k, k, 0, 1 \,\big|\, n_f, n_f - 1 - k, k, 0, 1\big) \Big] \\
&\times \mathcal{F}\, \mu\big(n_f, n_f - 1 - k, k, 0, 1\big)\,.
\end{aligned}
$$

$$
\begin{aligned}
\text{inflow}\big(n_f, n_f - 1 - k, k, 0, 1\big) = \mathcal{F} \sum_{n=n_f-1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} &\Bigg[\, \mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(n_f, n_f - 1 - k, k, 0, 1 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big) \\
&+ \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&\times \mathbb{P}\big(n_f, n_f - 1 - k, k, 0, 1 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&+ \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \\
&\times \mathbb{P}\big(n_f, n_f - 1 - k, k, 0, 1 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \Bigg] \\
&- \mathcal{F}\, \mu\big(n_f, n_f - 1 - k, k, 0, 1\big)\, \mathbb{P}\big(n_f, n_f - 1 - k, k, 0, 1 \,\big|\, n_f, n_f - 1 - k, k, 0, 1\big)\,.
\end{aligned}
$$

iv) $\mathcal{N}=(1,1,0,0,0)$

$$\text{outflow}\big(1,1,0,0,0\big) = \Big[\,1-\mathbb{P}\big(1,1,0,0,0\,\big|\,1,1,0,0,0\big)\,\Big]\,\mathcal{F}\,\mu\big(1,1,0,0,0\big)\,.$$

$$\begin{aligned}\text{inflow}(1,1,0,0,0) = {}& \mathcal{E}\,x_e\,\mu(\Delta^3)\frac{1}{2}(1-z^3) + \mathcal{F}\sum_{n=1}^{\overline{n}_f}\sum_{\widetilde{k}=0}^{n}\Bigg[\,\mu\big(n,n-\widetilde{k},\widetilde{k},0,0\big)\,\mathbb{P}\big(1,1,0,0,0\,\big|\,n,n-\widetilde{k},\widetilde{k},0,0\big)\\
&+\mu\big(n,n-1-\widetilde{k}\,I_{\{n>1\}},\widetilde{k}\,I_{\{n>1\}},1,0\big)\\
&\times\mathbb{P}\big(1,1,0,0,0\,\big|\,n,n-1-\widetilde{k}\,I_{\{n>1\}},\widetilde{k}\,I_{\{n>1\}},1,0\big)\\
&+\mu\big(n,n-1-\widetilde{k}\,I_{\{n>1\}},\widetilde{k}\,I_{\{n>1\}},0,1\big)\\
&\times\mathbb{P}\big(1,1,0,0,0\,\big|\,n,n-1-\widetilde{k}\,I_{\{n>1\}},\widetilde{k}\,I_{\{n>1\}},0,1\big)\Bigg]\\
&-\mathcal{F}\,\mu\big(1,1,0,0,0\big)\,\mathbb{P}\big(1,1,0,0,0\,\big|\,1,1,0,0,0\big)\,.\end{aligned}$$

v) $\mathcal{N}=(1,0,1,0,0)$

$$\text{outflow}\big(1,0,1,0,0\big) = \Big[\,1-\mathbb{P}\big(1,0,1,0,0\,\big|\,1,0,1,0,0\big)\,\Big]\,\mathcal{F}\,\mu\big(1,0,1,0,0\big)\,.$$

$$\begin{aligned}\text{inflow}(1,0,1,0,0) = {}& \mathcal{E}\,x_e\,\mu(\Delta^4)\,\left(1-\frac{1}{2}z^4\right) + \mathcal{F}\sum_{n=1}^{\overline{n}_f}\sum_{\widetilde{k}=0}^{n}\Bigg[\,\mu\big(n,n-\widetilde{k},\widetilde{k},0,0\big)\,\mathbb{P}\big(1,0,1,0,0\,\big|\,n,n-\widetilde{k},\widetilde{k},0,0\big)\\
&+\mu\big(n,n-1-\widetilde{k}\,I_{\{n>1\}},\widetilde{k}\,I_{\{n>1\}},1,0\big)\\
&\times\mathbb{P}\big(1,0,1,0,0\,\big|\,n,n-1-\widetilde{k}\,I_{\{n>1\}},\widetilde{k}\,I_{\{n>1\}},1,0\big)\\
&+\mu\big(n,n-1-\widetilde{k}\,I_{\{n>1\}},\widetilde{k}\,I_{\{n>1\}},0,1\big)\\
&\times\mathbb{P}\big(1,0,1,0,0\,\big|\,n,n-1-\widetilde{k}\,I_{\{n>1\}},\widetilde{k}\,I_{\{n>1\}},0,1\big)\Bigg]\\
&-\mathcal{F}\,\mu\big(1,0,1,0,0\big)\,\mathbb{P}\big(1,0,1,0,0\,\big|\,1,0,1,0,0\big)\,.\end{aligned}$$

vi) $\mathcal{N}=(1,0,0,1,0)$

$$\text{outflow}\big(1,0,0,1,0\big) = \Big[\,1-\mathbb{P}\big(1,0,0,1,0\,\big|\,1,0,0,1,0\big)\,\Big]\,\mathcal{F}\,\mu\big(1,0,0,1,0\big)\,.$$

$$\begin{aligned}\text{inflow}(1,0,0,1,0) = {}& \mathcal{E}\,x_e\,\mu(\Delta^1) + \mathcal{F}\sum_{n=1}^{\overline{n}_f}\sum_{\widetilde{k}=0}^{n}\Bigg[\,\mu\big(n,n-\widetilde{k},\widetilde{k},0,0\big)\,\mathbb{P}\big(1,0,0,1,0\,\big|\,n,n-\widetilde{k},\widetilde{k},0,0\big)\\
&+\mu\big(n,n-1-\widetilde{k}\,I_{\{n>1\}},\widetilde{k}\,I_{\{n>1\}},1,0\big)\times\mathbb{P}\big(1,0,0,1,0\,\big|\,n,n-1-\widetilde{k}\,I_{\{n>1\}},\widetilde{k}\,I_{\{n>1\}},1,0\big)\\
&+\mu\big(n,n-1-\widetilde{k}\,I_{\{n>1\}},\widetilde{k}\,I_{\{n>1\}},0,1\big)\times\mathbb{P}\big(1,0,0,1,0\,\big|\,n,n-1-\widetilde{k}\,I_{\{n>1\}},\widetilde{k}\,I_{\{n>1\}},0,1\big)\Bigg]\end{aligned}$$

$$- \mathcal{F}\, \mu\big(1,0,0,1,0\big)\, \mathbb{P}\big(1,0,0,1,0 \,\big|\, 1,0,0,1,0\big)\, .$$

vii) $\mathcal{N} = (1,0,0,0,1)$

$$\text{outflow}\big(1,0,0,0,1\big) = \Big[\, 1 - \mathbb{P}\big(1,0,0,0,1 \,\big|\, 1,0,0,0,1\big)\, \Big]\, \mathcal{F}\, \mu\big(1,0,0,0,1\big)\, .$$

$$\begin{aligned}
\text{inflow}(1,0,0,0,1) = \;& \mathcal{E}\, x_e\, \mu(\Delta^2)\,(1-z^2) + \mathcal{F} \sum_{n=1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} \Bigg[\, \mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(1,0,0,0,1 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big) \\
& + \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
& \times \mathbb{P}\big(1,0,0,0,1 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
& + \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \\
& \times \mathbb{P}\big(1,0,0,0,1 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \Bigg] \\
& - \mathcal{F}\, \mu\big(1,0,0,0,1\big)\, \mathbb{P}\big(1,0,0,0,1 \,\big|\, 1,0,0,0,1\big)\, .
\end{aligned}$$

A.5.1 Number of points in technology gap portfolio composition distribution

Let's denote $N(n_f)$ as the number of variations for a technology gap portfolio composition with n_f product lines, $\Big(n_f, n_f^1, n_f^2, n_f^3, n_f^4\Big)$, where $n_f = \sum_{\ell=1}^{4} n_f^\ell$, $n_f^3, n_f^4 \in \{0,1\}$, and $n_f^3 = n_f^4 = 1$ is not possible.

Let's denote $\widetilde{N}(n_f)$ as the number of variations for a technology gap portfolio composition with n_f product lines with no product line that has Δ^3 or Δ^4, $\Big(n_f, n_f^1, n_f^2, 0, 0\Big)$. Then:

$$N(n_f) = \widetilde{N}(n_f) + 2\widetilde{N}(n_f - 1)\, ,$$

as

$$\big(n_f, n_f^1, n_f^2, 1, 0\big) = \big(n_f - 1, n_f^1, n_f^2, 0, 0\big) + (1,0,0,1,0)\, ,$$

and

$$\big(n_f, n_f^1, n_f^2, 0, 1\big) = \big(n_f - 1, n_f^1, n_f^2, 0, 0\big) + (1,0,0,0,1)\, ,$$

and $\widetilde{N}(n_f) = n_f + 1$, $N(n_f) = 3\, n_f + 1$. Thus, for a maximum number of product line individual firm can have, $\overline{n}_f$, the total number of points in the technology gap portfolio composition distribution is

$$N_{\text{total}} = \sum_{n_f=1}^{\overline{n}_f} \big(3n_f + 1\big) = \frac{\big(3\overline{n}_f + 5\big)\, \overline{n}_f}{2}\, .$$

B Simple Three-Period Heterogeneous Innovation Model

To understand firms' incentives for internal and external innovation and derive empirically testable model predictions, we will consider a three-period economy with two product markets and three firms. In period 0, the economy starts with two product markets, market 1 and 2, with initial market-specific technologies denoted as $q_{1,0}$, and $q_{2,0}$, respectively. There are two firms in play, firm A and B. Product market 1 is given to firm A and is ready for production. Firm A is also endowed with an initial probability of internally innovating product 1, represented as $z_{1,0}$. Firm B, on the other hand, is provided with only a probability of externally innovating product 2, denoted as $x_{2,0}$. Thus, firm B can start operations and production in period 1 but not in period 0. If external innovation fails, then firm B still keeps market 2 but produces with the initial product quality $q_{2,0}$. Thus, at the beginning of period 1, product qualities in the two markets are equal to:

$$q_{1,1} = \begin{cases} \lambda\, q_{1,0} & \text{with probability } z_{1,0} \\ q_{1,0} & \text{with probability } 1 - z_{1,0}\ , \end{cases}$$

and

$$q_{2,1} = \begin{cases} \eta\, q_{2,0} & \text{with probability } x_{2,0} \\ q_{2,0} & \text{with probability } 1 - x_{2,0}\ . \end{cases}$$

where $\lambda^2 > \eta > \lambda > 1$ represent the innovation step sizes.

In period 1, the main period of interest, there is an outside firm (potentially from a foreign country) engaged in external innovation with the aim of taking over the two product markets in period 2. The success of the outside firm in external innovation is determined by the probability x_1^e for each product market. Additionally, there is a news shock about the period 2 profit (potentially including an increase in foreign demand) announced in period 1. Subsequently, the two incumbent firms produce using their given technologies, invest in internal innovation to improve the quality of their own products, and invest in external innovation to take over the other firm's product market. At the beginning of period 2, all innovation outcomes are realized. Following this, technological competition in each product market takes place, and only the firm with the highest technology in each product market produces. The economy ends after period 2.

In period 1, incumbent firm $i \in \{A, B\}$ invests $R_{j,1}^{in}$ in internal innovation, $j \in \{1, 2\}$ (e.g., for $i = A$, $j = 1$), resulting in a success probability $z_{j,1}$. The R&D production function is as follows:

$$z_{j,1} = \left(\frac{R_{j,1}^{in}}{\widehat{\chi} q_{j,1}} \right)^{\frac{1}{2}} .$$

Successful internal innovation increases the next-period product quality by $\lambda > 1$. Thus, the period 2 product quality for firm i becomes

$$q_{j,2}^i = \begin{cases} \lambda\, q_{j,1} & \text{with probability } z_{j,1} \\ q_{j,1} & \text{with probability } 1 - z_{j,1}\ . \end{cases}$$

Similarly, firm i invests $R^{ex}_{-j,1}$ to learn the period 0 technology used by firm $-i \neq i$, which governs the success probability of external innovation $x_{-j,1}$ the R&D production function as follows:

$$x_{-j,1} = \left(\frac{R^{ex}_{-j,1}}{\widetilde{\chi} q_{-j,0}} \right)^{\frac{1}{2}} ,$$

where $-j$ is owned by $-i$. Successful external innovation increases product quality relative to the past-period quality by $\eta > 1$. Thus, product $-j$'s quality in period 2 for firm i becomes

$$q^i_{-j,2} = \begin{cases} \eta\, q_{-j,0} & \text{with probability } x_{-j,1} \\ \varnothing & \text{with probability } 1 - x_{-j,1} \, , \end{cases}$$

where $\varnothing$ means firm i has failed to acquire any production technology for product $-j$.

B.1 Optimal Innovation Decisions and Theoretical Predictions

Assume that in a given product market j and period t, firms receive instantaneous profit of $\pi_{j,t}\, q_{j,t}$ where $q_{j,t}$ is the product quality and $\pi_{j,t}$ is a market-period-specific constant known to firms before each period begins. Because there are only two products, incumbents and the outside firm can perform external innovation on the same product. To main simplicity in the model, we further assume that the outside firm can do external innovation only if an incumbent fails to do external innovation, following the approach of Garcia-Macia et al. (2019). Then the profit maximization problem for firm i that has product market j with quality $q_{j,1}$ in period 1 can be written as:

$$V(q_{j,1}) = \max_{\{z_{j,1},\, x_{-j,1}\}} \left\{ \begin{array}{l} \pi q_{j,1} - \widehat{\chi}(z_{j,1})^2 q_{j,1} - \widetilde{\chi}(x_{-j,1})^2 q_{-j,0} \\[1em] +(1 - x_{j,1})(1 - x^e_1)\Big[(1 - z_{j,1})\pi_{j,2} q_{j,1} + z_{j,1}\pi_{j,2}\lambda q_{j,1}\Big] \\[1em] +\big[x_{j,1} + (1 - x_{j,1})\, x^e_1\big] \Big[z_{j,1}\pi_{j,2}\, \lambda\, q_{j,1}\, \mathcal{I}_{\{\lambda q_{j,1} \,>\, \eta q_{j,0}\}} \\ \qquad\qquad\qquad\qquad\qquad + \frac{1}{2}(1 - z_{j,1})\pi_{j,2} q_{j,1}\, \mathcal{I}_{\{q_{j,1} \,=\, \eta q_{j,0}\}} \Big] \\[1em] +x_{-j,1}\Big[(1 - z_{-j,1})\, \pi_{-j,2}\, \eta\, q_{-j,0}\, \mathcal{I}_{\{\eta q_{-j,0} \,>\, q_{-j,1}\}} \\ \qquad\qquad + z_{-j,1}\, \pi_{-j,2}\, \eta\, q_{-j,0}\, \mathcal{I}_{\{\eta q_{-j,0} \,>\lambda q_{-j,1}\}} \\ \qquad\qquad + \frac{1}{2}(1 - z_{-j,1})\pi_{-j,2}\eta q_{-j,0}\, \mathcal{I}_{\{\eta q_{-j,0} \,=\, q_{-j,1}\}} \\ \qquad\qquad + \frac{1}{2} z_{-j,1}\pi_{-j,2}\eta q_{-j,0}\, \mathcal{I}_{\{\eta q_{-j,0} \,=\, \lambda q_{-j,1}\}} \Big] \end{array} \right\} ,$$

where $\mathcal{I}_{\{\cdot\}}$ is an indicator function that captures the possible relationships between the two technologies among the three firms in period 2 in a given market. The first line shows the period 1 profit net of the total R&D cost. The second line represents the incumbent's period 2 expected profit from market j when the other incumbent and the outside firm fail to externally innovate the market j technology. The third and the fourth line represent the period 2 expected profit from market j when one of the two other firms succeeds in externally innovating the market j technology. The fifth to eighth lines represent the period 2 expected profit from market $-j$ when firm i succeeds in externally innovating the market $-j$ technology. The terms following $\frac{1}{2}$ account for cases in which two firms can produce the same quality product, triggering a coin-toss tiebreaker rule.

The interior solutions to this problem are

$$z^*_{j,1} = \begin{cases} \dfrac{\pi_{j,2}}{2\,\widehat{\chi}}\,(\lambda-1)(1-x^*_{j,1})(1-x^e_1) & \text{, when } q_{j,1} = q_{j,0} \\ \dfrac{\pi_{j,2}}{2\,\widehat{\chi}}\,\left[\lambda-(1-x^*_{j,1})(1-x^e_1)\right] & \text{, when } q_{j,1} = \lambda\, q_{j,0} \\ \dfrac{\pi_{j,2}}{2\,\widehat{\chi}}\,\left[\lambda-\dfrac{1}{2}-\dfrac{1}{2}(1-x^*_{j,1})(1-x^e_1)\right] & \text{, when } q_{j,1} = \eta\, q_{j,0} \end{cases}$$

and

$$x^*_{-j,1} = \begin{cases} \dfrac{\eta\,\pi_{-j,2}}{2\,\widetilde{\chi}} & \text{, when } q_{-j,1} = q_{-j,0} \\ \dfrac{\eta\,\pi_{-j,2}}{2\,\widetilde{\chi}}\,(1-z^*_{-j,1}) & \text{, when } q_{-j,1} = \lambda\, q_{-j,0} \\ \dfrac{\eta\,\pi_{-j,2}}{2\,\widetilde{\chi}}\,\dfrac{1}{2}(1-z^*_{-j,1}) & \text{, when } q_{-j,1} = \eta\, q_{-j,0}\,. \end{cases}$$

The above results show that the firm's optimal innovation decisions depend on the (expected) future profit, the technology gap in both its own market and the other firm's market, and other firms' internal and external innovation decisions. From these interior solutions, I draw the following results:

Proposition B.1. *For each $q_{j,1}$ and for $\lambda^2 > \eta > \lambda > 1$, we can order internal innovation intensities as*

$$z^*_{j,1}\Big|_{q_{j,1}=\lambda q_{j,0}} \quad > \quad z^*_{1,1}\Big|_{q_{j,1}=\eta q_{j,0}} \quad > \quad z^*_{j,1}\Big|_{q_{j,1}=q_{j,0}}\,.$$

Furthermore,

$$\left.\frac{\partial z^*_{j,1}}{\partial x^e_1}\right|_{q_{j,1}=\lambda q_{j,0}} \quad > \quad \left.\frac{\partial z^*_{j,1}}{\partial x^e_1}\right|_{q_{j,1}=\eta q_{j,0}} \quad > \quad 0 \quad > \quad \left.\frac{\partial z^*_{j,1}}{\partial x^e_1}\right|_{q_{j,1}=q_{j,0}}\,.$$

Proof: See Appendix B.2.1

The second part of proposition B.1 implies that firms with no local technology gap reduce their internal innovation investment when faced with a higher probability of creative destruction in their own markets. This occurs because they cannot increase the probability of protecting their markets by improving their products through internal innovation. On the other hand, if a firm has a substantial technological advantage, it does not significantly increase its internal innovation investment in response to outsiders' investment in external innovation, as the probability of losing its own product market is small. In the intermediate case, firms increase their internal innovation investment more strongly in response to outsiders' external innovation because they can lower the probability of losing their market by doing so.

Higher innovation in period 0 increases the probability of having a high local technology gap in period 1 and this helps firms to protect their markets. To understand how past innovation intensity affects the firm's current internal innovation decision when confronted with a higher probability of encountering a competitor, x_1^e, define the expected value of internal innovation intensity in period 1 as:

$$\overline{z}_1^* = z_{1,1}^*\big|_{q_{1,1}=q_{1,0}} \frac{1}{2}(1-z_{1,0}) \;+\; z_{2,1}^*\big|_{q_{2,1}=q_{2,0}} \frac{1}{2}(1-x_{2,0}) \;+\; z_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} \frac{1}{2}z_{1,0} \;+\; z_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} \frac{1}{2}x_{2,0}\,,$$

where $\frac{1}{2}$ comes from the fact that there are two products. Then, proposition B.1 provides the following result:

Corollary B.1 (Market-Protection Effect). *The impact of period 0 innovation intensities, $z_{1,0}$ and $x_{2,0}$ on expected internal innovation in period 1 can be characterized as follows:*

$$\frac{\partial \overline{z}_1^*}{\partial x_1^e \partial z_{1,0}} > 0\,, \text{ and } \frac{\partial \overline{z}_1^*}{\partial x_1^e \partial x_{2,0}} > 0\,.$$

Proof: See Appendix B.2.2

Corollary B.1 suggests that intensive innovation in the previous period prompts firms to enhance their response to higher product market competition through increased internal innovation. As the optimal decision rule indicates, firms' external innovation decision also relies on the past innovation decisions of other firms:

Proposition B.2. *For each $q_{j,1}$ and for $\lambda^2 > \eta > \lambda > 1$, we can order external innovation intensities as follows:*

$$x_{j,1}^*\big|_{q_{j,1}=q_{j,0}} \quad > \quad x_{j,1}^*\big|_{q_{j,1}=\lambda q_{j,0}} \quad > \quad x_{j,1}^*\big|_{q_{j,1}=\eta q_{j,0}}$$

Furthermore,

$$\frac{\partial x_{j,1}^*}{\partial x_1^e}\bigg|_{q_{j,1}=q_{j,0}} = 0\,, \quad \frac{\partial x_{j,1}^*}{\partial x_1^e}\bigg|_{q_{j,1}=\lambda q_{j,0}} < 0\,, \text{ and } \quad \frac{\partial x_{j,1}^*}{\partial x_1^e}\bigg|_{q_{j,1}=\eta q_{j,0}} < 0\,.$$

Proof: See Appendix B.2.1

Proposition B.2 implies that firms do less external innovation if other firms have a higher technology advantage, as it becomes more difficult to take over their markets through external innovation. For product markets with a technological barrier (local technology gap > 1), firms also lower their external innovation if the outside firm does more external innovation, as incumbents in these markets will respond by doing more internal innovation with defensive motive (proposition B.1). To understand how the past innovation intensity of other firms affects a firm's current external innovation decision, define the expected value of external innovation intensity in period 1 as

$$\overline{x}_1^* = x_{1,1}^*\big|_{q_{1,1}=q_{1,0}} \frac{1}{2}(1-z_{1,0}) \;+\; x_{2,1}^*\big|_{q_{2,1}=q_{2,0}} \frac{1}{2}(1-x_{2,0}) \;+\; x_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} \frac{1}{2}z_{1,0} \;+\; x_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} \frac{1}{2}x_{2,0}\,.$$

Then, the first part of proposition B.2 implies the following:

Corollary B.2 (Technological Barrier Effect). *For a given technology $q_{j,1}$ and period 0 innovation intensities, $z_{1,0}$ and $x_{2,0}$, we have*

$$\frac{\partial \overline{x}_1^*}{\partial z_{1,0}} < 0\,, \text{ and } \frac{\partial \overline{x}_1^*}{\partial x_{2,0}} < 0\,.$$

Proof: See Appendix B.2.3

Corollary B.2 indicates that higher technology levels in other markets, resulting from previous innovation, act as an effective technological barrier, making it challenging for outside firms to take over others' product market. This reduces firms' incentive for external innovation. Because innovation is forward looking, changes in future profit π' are an important factor affecting current period innovation intensity. Proposition B.3 summarizes this:

Proposition B.3 (Ex-post Schumpeterian Effect)**.** *Given the expected profit $\pi_{j,2}$ in period 2, we obtain:*

$$\frac{\partial z^*_{j,1}}{\partial \pi_{j,2}} > 0\,, \quad \forall\, q_{j,1} \text{ , and} \qquad \frac{\partial x^*_{j,1}}{\partial \pi_{j,2}} > 0\,, \quad \text{for } q_{j,1} = q_{j,0}\,.$$

The signs for $\frac{\partial x^*_{j,1}}{\partial \pi_{j,2}}$ for other local technology gaps are ambiguous. Proof: See Appendix B.2.4

Proposition B.3 implies that any factor that affects future profits may influence firms' internal and external innovation. These factors include changes in market size (such as opportunities to access foreign markets), variations in input costs, and changes in future survival probability. Specifically, an increase in expected profit from one's own market encourages firms to intensify their internal innovation efforts. However, the impact of an increase in expected profit in other markets on firms' external innovation is ambiguous for cases where the local technology gap is greater than 1. This ambiguity arises because incumbents in these markets boost their internal innovation in response to increasing expected profit, allowing them to protect their markets. In cases where the local technology gap is equal to 1, incumbents cannot protect their markets through internal innovation. Thus, an increase in expected future profit unambiguously promotes external innovation in this scenario. These findings highlight the diverse factors influencing internal, external, and overall innovation.

B.2 Proofs for the Simple Model

B.2.1 Proof for Proposition B.1

Proof. The first part of proposition B.1 follows from simple algebra. The second part is proved as follows. For $q_{j,1} = q_{j,0}$, we have

$$\frac{\partial z_{j,1}}{x^e_1} = -\frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda - 1)\left[(1 - x_{j,1}) + (1 - x^e_1)\frac{\partial x_{j,1}}{\partial x^e_1}\right]\,,$$

and

$$\frac{\partial x_{j,1}}{\partial x^e_1} = 0\,.$$

Thus, the following is obtained:

$$\frac{\partial z_{j,1}}{\partial x^e_1} = -\frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda - 1)(1 - x_{j,1}) < 0\,.$$

For $q_{j,1} = \lambda q_{j,0}$, we have

$$\frac{\partial z_{j,1}}{\partial x_1^e} = \frac{\pi_{j,2}}{2\widehat{\chi}}\left[1 - x_{j,1} + (1 - x_1^e)\frac{\partial x_{j,1}}{\partial x_1^e}\right] ,$$

and

$$\frac{\partial x_{j,1}}{\partial x_1^e} = -\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x_1^e} .$$

Thus, the following holds:

$$\frac{\partial z_{j,1}}{\partial x_1^e} = (1 - x_{j,1})\left[\frac{2\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{2\widetilde{\chi}}(1 - x_1^e)\right]^{-1} > 0 ,$$

hence

$$\frac{\partial x_{j,1}}{\partial x_1^e} = -\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x_1^e} < 0 .$$

For $q_{j,1} = \eta q_{j,0}$, we have

$$\frac{\partial z_{j,1}}{\partial x_1^e} = \frac{\pi_{j,2}}{2\widehat{\chi}}\frac{1}{2}\left[1 - x_{j,1} + (1 - x_1^e)\frac{\partial x_{j,1}}{\partial x_1^e}\right] ,$$

and

$$\frac{\partial x_{j,1}}{\partial x_1^e} = -\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{1}{2}\frac{\partial z_{j,1}}{\partial x_1^e} .$$

This derives the following:

$$\frac{\partial z_{j,1}}{\partial x_1^e} = (1 - x_{j,1})\left[\frac{4\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{4\widetilde{\chi}}(1 - x_1^e)\right]^{-1} > 0 ,$$

hence

$$\frac{\partial x_{j,1}}{\partial x_1^e} = -\frac{1}{2}\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x_1^e} < 0 .$$

From $x_{j,1}^*$, given that $\frac{\eta\pi_{j,2}}{2\widetilde{\chi}} \in (0,1)$, if we impose a parameter restriction $4\widehat{\chi} > \pi_{j,2}$, the following holds:

$$\frac{4\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{4\widetilde{\chi}}(1 - x_1^e) > \frac{2\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{2\widetilde{\chi}}(1 - x_1^e) .$$

Therefore, we get $\left.\frac{\partial z_{j,1}^*}{\partial x_1^e}\right|_{q_{j,1}=\lambda q_{j,0}} > \left.\frac{\partial z_{j,1}^*}{\partial x_1^e}\right|_{q_{j,1}=\eta q_{j,0}}$ $\square$

B.2.2 Proof of Corollary B.1

Proof. From $\overline{z}_1^*$, we know that

$$\frac{\partial \overline{z}_1^*}{\partial z_{1,0}} = \frac{1}{2}\left(z_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} - z_{1,1}^*\big|_{q_{1,1}=q_{1,0}} \right) > 0\,,$$

and

$$\frac{\partial \overline{z}_1^*}{\partial x_{2,0}} = \frac{1}{2}\left(z_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} - z_{2,1}^*\big|_{q_{2,1}=q_{2,0}} \right) > 0\,,$$

where the signs of the two derivatives follow from proposition B.1. Then, the results follow from proposition B.1. □

B.2.3 Proof of Corollary B.2

Proof. From $\overline{x}_1^*$, we have

$$\frac{\partial \overline{x}_1^*}{\partial z_{1,0}} = \frac{1}{2}\left(x_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} - x_{1,1}^*\big|_{q_{1,1}=q_{1,0}} \right) < 0\,,$$

and

$$\frac{\partial \overline{x}_1^*}{\partial x_{2,0}} = \frac{1}{2}\left(x_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} - x_{2,1}^*\big|_{q_{2,1}=q_{2,0}} \right) < 0\,,$$

where the signs for the two derivatives follow from proposition B.2 □

B.2.4 Proof of Proposition B.3

Proof. For $q_{j,1} = q_{j,0}$,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}(\lambda-1)(1-x_{j,1})(1-x_1^e) - \frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda-1)(1-x_1^e)\frac{\partial x_{j,1}}{\partial \pi_{j,2}}\,,$$

and

$$\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{\eta}{2\widetilde{\chi}}$$

Thus,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}(\lambda-1)(1-2x_{j,1})(1-x_1^e)\,,$$

and this is positive iff $x_{j,1} < \frac{1}{2}$. Therefore, $\frac{\partial x_{j,1}}{\partial \pi_{j,2}} > 0$ is proved unambiguously.

For $q_{j,1} = \lambda q_{j,0}$,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}\left[\lambda - (1 - x_{j,1})(1 - x_1^e)\right] + \frac{\pi_{j,2}}{2\widehat{\chi}}(1 - x_1^e)\frac{\partial x_{j,1}}{\partial \pi_{j,2}} ,$$

and

$$\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{x_{j,1}}{\pi_{j,2}} - \frac{\eta \pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial \pi_{j.2}} .$$

Thus,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \left[\lambda - (1 - 2x_{j,1})(1 - x_1^e)\right]\left[2\widehat{\chi} + \frac{\eta(\pi_{j,2})^2}{2\widetilde{\chi}}(1 - x_1^e)\right]^{-1} ,$$

and this is positive unambiguously. The sign for $\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ is ambiguous.

For $q_{j,1} = \eta q_{j,0}$,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}\left[\lambda - \frac{1}{2} - \frac{1}{2}(1 - x_{j,1})(1 - x_1^e)\right] + \frac{\pi_{j,2}}{2\widehat{\chi}}\frac{1}{2}(1 - x_1^e)\frac{\partial x_{j,1}}{\partial \pi_{j,2}} ,$$

and

$$\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{\eta}{2\widetilde{\chi}}\frac{1}{2}(1 - z_{j,1}) - \frac{\eta \pi_{j,2}}{2\widetilde{\chi}}\frac{1}{2}\frac{\partial z_{j,1}}{\partial \pi_{j,2}} .$$

Thus,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \left[\lambda - \frac{1}{2} - \frac{1}{2}(1 - 2x_{j,1})(1 - x_1^e)\right]\left[2\widehat{\chi} + \frac{\eta(\pi_{j,2})^2}{2\widetilde{\chi}}\frac{1}{4}(1 - x_1^e)\right]^{-1} ,$$

and this is positive unambiguously. The sign for $\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ is ambiguous. $\square$

C Data Appendix

C.1 Summary Statistics

Table C1: Foreign Competition Shock Related Measures

	NTR gap	Dnstream NTR g.	Upstream NTR g.	NTR rate	Non-NTR r.
Mean	0.291	0.138	0.203	0.027	0.303
(Std. dev.)	(0.127)	(0.060)	(0.073)	(0.022)	(0.134)
cov(, NTR gap)		0.485	0.434	0.412	0.969
cov(, Up. NTR g.)		0.204			

Table C2: Industry-level NTR gap

	NTR gap, main industry
Mean	0.336
(Std. dev.)	(0.116)
cov(, NTR gap)	0.86

Table C3: The Whole Universe of Patenting Firms vs. Regression Sample in 1992

	All patenting firms	Regression sample
Average number of patents	6.15	8.86
	(19.46)	(24.10)
Average self-citation rate	0.0434	0.0540
	(0.0899)	(0.0941)
Innovation intensity	0.055	0.093
	(0.25)	(0.33)
Number of industries operating	2.34	5.43
	(3.67)	(6.94)
Employment	511.7	1988.0
	(1869.0)	(3835.0)
Patent stock	6.45	35.22
	(26.61)	(64.37)
Employment growth	0.07	0.06
	(0.60)	(0.40)
Firm age	12.33	15.65
	(6.76)	(9.42)
7yr patent growth		-0.854
		(1.312)
7yr self-citation ratio growth		0.356
		(1.322)
Number of firms	26,500	3,100

Notes: Innovation intensity in 2000 is 0.183(0.58), the seven-year patent growth in 2000 is -1.07(1.207), and the seven-year self-citation ratio growth in 2000 is 0.282(1.304).

Table C4: Export Share of Total Value of Shipments (CMF exporters)

	1992	2002	2007
Avg. of firm-level exp/vship	4.99%	5.27%	6.41%
Avg. of firm-level CN exp/vship	0.70%	0.89%	1.17%
Aggregate-level exp/vship	7.76%	9.29%	10.46%
Aggregate-level CN exp/vship	0.19%	0.38%	0.64%

Table C5: Share of Exporters (LBD firms)

Year	1992	2002	2007
Share of exporters	15.90%	22.10%	24.00%
Share of firms exporting to CN	0.60%	2.30%	4.00%

C.2 Overall and Market-Protection Effect (Full Tables)

Table C6: Overall Effect

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap $\times$ Post	0.226	0.067	0.025	0.045
	(0.230)	(0.275)	(0.260)	(0.291)
NTR gap	-2.222***	0.392	1.104***	-0.058
	(0.372)	(0.409)	(0.317)	(0.390)
Past 5yr Δpat in own tech.		0.165**		0.265***
		(0.084)		(0.083)
Log employment		0.160***		-0.026**
		(0.011)		(0.012)
Firm age		-0.006**		-0.011***
		(0.002)		(0.002)
NTR rate		-2.456		1.248
		(1.672)		(2.220)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	no	baseline	no	baseline

Notes: The baseline controls include the past five-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, and a dummy for publicly traded firms. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C7: Market-Protection Effect

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	0.238	0.071	-0.075	-0.062
	(0.237)	(0.283)	(0.257)	(0.291)
× Innovation intensity	0.077	-0.054	0.732**	0.795***
	(0.231)	(0.242)	(0.299)	(0.277)
NTR gap	-2.206***	0.418	1.101***	-0.005
	(0.375)	(0.412)	(0.315)	(0.394)
× Innovation intensity	-0.226	-0.161	-0.198	-0.390
	(0.158)	(0.184)	(0.231)	(0.236)
Post × Innovation intensity	-0.053	0.040	-0.179*	-0.202**
	(0.070)	(0.079)	(0.095)	(0.087)
Innovation intensity	0.080*	0.029	0.059	0.088
	(0.048)	(0.051)	(0.070)	(0.068)
Past 5yr Δpat in own tech.		0.164*		0.265***
		(0.084)		(0.083)
Log employment		0.161***		-0.025**
		(0.011)		(0.012)
Firm age		-0.005**		-0.011***
		(0.002)		(0.002)
NTR rate		-2.619		1.024
		(1.683)		(2.224)
× Innovation intensity		0.690		0.625
		(0.531)		(0.501)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	no	baseline	no	baseline

Notes: The baseline controls include the past five-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, and a dummy for publicly traded firms. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

C.3 Firm Growth and Two Types of Innovation

Table C8: Real Effect of Innovation on Employment Growth, Industry and Product Added

	LBD firms		CMF firms
	ΔEmployment	Log nb. of industries added	Log nb. of products added
	(1)	(2)	(3)
Log nb. of patents	0.036***	0.102***	0.083***
	(0.010)	(0.011)	(0.013)
Avg. self-citation	-0.256**	-0.158**	-0.286***
	(0.109)	(0.079)	(0.093)
Log payroll	-0.027***	0.113***	0.149***
	(0.008)	(0.006)	(0.005)
Firm age	-0.004**	0.002	-0.002
	(0.002)	(0.002)	(0.002)
Innovation intensity	-0.004	-0.006**	0.009
	(0.003)	(0.003)	(0.008)
Past 5yr Δpat in own tech.	0.001	-0.000	-0.000
	(0.001)	(0.001)	(0.000)
Observations	5,400	5,400	5,700
Fixed effects	jp	jp	jp

Notes: The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions. Robust standard errors adjusted for clustering at the firm-level are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Akcigit and Kerr (2018) show that internal innovation contributes less to firm employment growth by using the LBD. Here, we replicate their result while including firm controls for the Census years: 1982, and 1992 and construct non-overlapping five-year first differences (DHS growth) by using the linked LBD-USPTO patent database. We estimate the following fixed-effect regression model:

$$\Delta Y_{ijt+5} = \beta_1 Pat_{ijt} + \beta_2 Internal_{ijt} + \mathbf{X}_{ijt}\,\gamma_1 + \delta_{jt+5} + \varepsilon_{ijt+5}.$$

For firm i in industry j, ΔY_{ijt+5} is the five-year DHS growth rate of i) firm employment growth from year t to $t+5$, and ii) the number of six-digit NAICS industries added. Pat_{ijt} is a log of the citation adjusted number of patents in year t, and $Internal_{ijt}$ is an citation-adjusted average self-citation ratio in year t. Firm and industry controls include firm age, log payroll, the past five-year U.S. patent growth in firms' own technology fields, innovation intensity, and a dummy for publicly traded firms. The regression is unweighted and standard errors are clustered on firm. The mean (and standard deviation) of the logged average number of patents is 1.284(1.125), and the average self-citation ratio is 0.050(0.101) (both citation-adjusted). Based on Akcigit and Kerr (2018) we expect β_1 to be positive and β_2 to be negative, as internal innovation contributes less to firm employment growth. We run the same regression model replacing the dependent variable with the number of products (seven-digit NAICS product codes) added for the CMF firms.

Column 1 shows that for average firms, creating one more logged number of patents is associated with a 3.6

percentage point increase in their employment growth. Since $\exp(1) \approx 2.718$, this means that creating one more patent is associated with a 1.32 percentage point (3.6/2.718) increase in their employment growth. Also, since average firms have an average self-citation ratio of 0.05, a 1% increase in the self-citation ratio is associated with a 0.0128 percentage point (-0.256$\times$0.05$\times$0.01$\times$100) decrease in their employment growth.

C.4 Parallel Pre-trend Assumption

Table C9: Parallel Pre-trend Test

	Overall	Market-protection	Overall	Market-protection
	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap	-0.397	-0.380	-0.554	-0.546
	(0.487)	(0.488)	(0.403)	(0.402)
$\times$ Innovation intensity		-0.195		-0.058
		(0.162)		(0.395)
NTR gap $\times$ $\mathcal{I}_{\{1992\}}$	0.523	0.500	0.252	0.259
	(0.355)	(0.362)	(0.294)	(0.290)
$\times$ Innovation intensity		0.092		-0.113
		(0.243)		(0.491)
Post $\times$ Innovation intensity		0.009		0.027
		(0.064)		(0.115)
Innovation intensity		0.036		0.022
		(0.033)		(0.082)
Past 5yr Δpat in own tech.	0.149	0.151	0.105	0.104
	(0.096)	(0.096)	(0.097)	(0.098)
Log employment	0.157***	0.156***	-0.047***	-0.047**
	(0.013)	(0.013)	(0.014)	(0.014)
Firm age	0.000	0.000	-0.009***	-0.009***
	(0.003)	(0.003)	(0.003)	(0.003)
Observations	5,000	5,000	5,000	5,000
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline

Notes: The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

C.5 Other Robustness Test

Table C10: Robustness Test for the Overall Response

	ΔPatents	ΔPatents	ΔPatents	ΔPatents	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite	ΔSelf-cite	ΔSelf-cite	ΔSelf-cite ΔSelf-cite	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
NTR gap × Post	0.074	0.059	0.023	0.118	0.075	0.067	0.030	0.048	0.081	0.114	0.149	0.166
	(0.276)	(0.276)	(0.276)	(0.271)	(0.272)	(0.272)	(0.291)	(0.292)	(0.290)	(0.290)	(0.288)	(0.287)
NTR gap	0.396	0.417	0.566	0.390	0.563	0.567	-0.067	-0.065	-0.200	-0.067	-0.206	-0.249
	(0.408)	(0.407)	(0.406)	(0.412)	(0.409)	(0.408)	(0.388)	(0.390)	(0.385)	(0.379)	(0.374)	(0.376)
Observations	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500
Fixed effects	j,p	j,p	j,p	j,p	j,p	j,p	j,p	j,p	j,p	j,p	j,p	j,p
Controls	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+

Notes: All columns augment the baseline set of controls with additional variables. Specifically, columns (1),(7) include the cumulative number of patents, column (2),(8) include firm payroll, column (3),(9) include the number of industries in which firms operate, column (4),(10) include the industry-level skill, capital intensities, column (5),(11) include the number of industries and the industry-level skill, capital intensities, column (6),(12) include the number of industries, the industry-level skill, capital intensities, a dummy for firms with total imports > 0, and a dummy for firms with total exports > 0. The estimates for industry (j) and the period (p) fixed effects, and the coefficients associated with the binary indicators are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C11: Robustness Test for the Market-Protection Effect

	ΔPatents	ΔPatents	ΔPatents	ΔPatents	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite	ΔSelf-cite	ΔSelf-cite	ΔSelf-cite ΔSelf-cite	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
NTR gap × Post	0.076	0.062	0.028	0.112	0.081	0.074	-0.078	-0.059	-0.026	0.007	0.042	0.063
	(0.283)	(0.284)	(0.284)	(0.278)	(0.279)	(0.280)	(0.290)	(0.291)	(0.289)	(0.287)	(0.285)	(0.285)
× Innovation intensity	-0.055	-0.037	-0.051	0.058	-0.055	-0.029	0.798***	0.789***	0.792***	0.789***	0.787***	0.777***
	(0.242)	(0.242)	(0.239)	(0.243)	(0.240)	(0.231)	(0.278)	(0.278)	(0.280)	(0.277)	(0.279)	(0.268)
Observations	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p
Controls	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+	baseline+

Notes: All columns augment the baseline set of controls with additional variables. Specifically, columns (1),(7) include the cumulative number of patents, column (2),(8) include firm payroll, column (3),(9) include the number of industries in which firms operate, column (4),(10) include the industry-level skill, capital intensities, column (5),(11) include the number of industries and the industry-level skill, capital intensities, column (6),(12) include the number of industries, the industry-level skill, capital intensities, a dummy for firms with total imports > 0, and a dummy for firms with total exports > 0. The estimates for industry (j) and the period (p) fixed effects, and the coefficients associated with the binary indicators are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C12: Foreign Competition Shock through I-O Linkages

	Overall	Market-protection	Overall	Market-protection
	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap $\times$ Post	-0.111	-0.111	-0.296	-0.424
	(0.331)	(0.342)	(0.356)	(0.355)
$\times$ Innovation intensity		-0.001		0.824***
		(0.337)		(0.288)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline+IO	baseline	baseline	baseline

Notes: The baseline set of controls is included along with the diff-in-diff terms for upstream and downstream sectors, respectively. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C13: Industry-level Tariff Measures

	Overall	Market-protection	Overall	Market-protection
	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap $\times$ Post	0.016	0.011	0.005	-0.001
	(0.249)	(0.249)	(0.261)	(0.261)
$\times$ Innovation intensity		-0.032		0.760***
		(0.229)		(0.272)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline
Weights for tariffs	major industry	major industry	major industry	major industry

Notes: Table reports results of OLS generalized difference-in-differences regressions in which industry-level tariff measures are used. The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C14: Weighted by Inverse Propensity Score

	Overall	Market-protection	Overall	Market-protection
	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap $\times$ Post	0.003	0.039	-0.394	-0.603
	(0.475)	(0.484)	(0.509)	(0.512)
$\times$ Innovation intensity		-0.045		0.893***
		(0.282)		(0.294)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline
Regression weights	inv. propens.	inv. propens.	inv. propens.	inv. propens.

Notes: Table reports results of OLS generalized difference-in-differences regressions in which observations are weighted by the inverse of the propensity scores from logit model (y = indicator for analysis sample). The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. For the sake of space, only the main coefficients are presented. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C15: Standard Error Clustring on Firms

	Overall	Market-protection	Overall	Market-protection
	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap $\times$ Post	0.067	0.071	0.045	-0.062
	(0.287)	(0.290)	(0.308)	(0.312)
$\times$ Innovation intensity		-0.054		0.795***
		(0.245)		(0.277)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline
se. cluster	firmid	firmid	firmid	firmid

Notes: Table reports results of OLS generalized difference-in-differences regressions in which robust standard errors are adjusted for clustering at the firm-level. The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. For the sake of space, only the main coefficients are presented. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table C16: Effect of Foreign Competition on Product Added

	Log number of products added	Log number of products added
	(1)	(2)
NTR gap $\times$ Post	-0.211***	-0.210***
	(0.069)	(0.069)
$\times$ Innovation intensity		-0.584***
		(0.210)
Observations	497,000	497,000
Fixed effects	j, p	j, p
Controls	baseline	baseline

Notes: The baseline set of controls is included. Estimates for industry-period (jp) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p $<$ 0.1, ** p $<$ 0.05, *** p $<$ 0.01.

D Solution Algorithm

In the model, $\{z^\ell\}_{\ell=1}^4$ are functions of $\overline{x}$; g is a function of $\overline{x}$, $\{z^\ell\}_{\ell=1}^4$, and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; x is a function of $\overline{x}$ and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; x_e is a function of $\overline{x}$ and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; and $\overline{x}$ is a function of $\mathcal{F}_d$, x, and x_e. Therefore, we can solve for an equilibrium of the model by iterating over the value for the aggregate creative destruction arrival rate $\overline{x}$.

i) Guess a value for $\overline{x}$ and the technology gap portfolio composition distribution $\mu(\mathcal{N})$, which imply the technology gap distribution $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ and the total mass of domestic firms $\mathcal{F}_d$.

ii) Using the guess for $\overline{x}$, compute $\{A_\ell\}_{\ell=1}^4$, and $\{z^\ell\}_{\ell=1}^4$.

iii) Using the guess for $\mu(\mathcal{N})$, $\{\mu(\Delta^\ell)\}_{\ell=1}^4$, and $\mathcal{F}_d$,

   a) Compute g, x, B, and x_e.

   b) Compute stationary $\mu_\infty(\mathcal{N})$ and $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$, based on the guess for $\mu(\mathcal{N})$, innovation decision rules, and the following law of motion

$$\mathcal{F}_{d,n+1}\,\mu_{n+1}(\mathcal{N}) = \mathcal{F}_{d,n}\,\mu_n(\mathcal{N}) + \text{inflow}_n(\mathcal{N}) - \text{outflow}_n(\mathcal{N})\,.$$

   c) Compute g_∞, x_∞, B_∞, and x_{e_∞}, with $\mu_\infty(\mathcal{N})$, and $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$.

iv) Obtain $\overline{x}' = \mathcal{F}_{d,\infty}\,x_\infty + \mathcal{E}_d\,x_{e_\infty}$.

v) If $\overline{x} \neq \overline{x}'$, set $\overline{x} = \overline{x}'$, and $\mu(\mathcal{N}) = \mu_\infty(\mathcal{N})$, use them as new guess, and return to ii).

vi) Repeat ii) through v) until the convergence of $\overline{x}$.

E Other Theoretical Results

Table E1: Aggregate Growth Rate Decomposition, Holding Mass Fixed

Description	Before	After	% Change
average growth rate (g, %)	1.049	1.059	0.96%
growth rate by outside firms (%)	0.155	0.172	10.84%
growth rate by domestic firms (%)	0.894	0.887	-0.78%
growth rate by domestic incumbents (%)	0.774	0.771	-0.39%
growth rate from domestic internal innovation (%)	0.544	0.546	0.29%
growth rate from domestic external innovation (%)	0.230	0.225	-2.01%
growth rate from domestic startups (%)	0.121	0.117	-3.29%

Table E2: Aggregate Growth Rate Decomposition, Low Creativity Economy, Holding Mass Fixed

Description	Before	After	% Change
average growth rate (g, %)	0.722	0.740	2.51%
growth rate by outside firms (%)	0.151	0.172	13.64%
growth rate by domestic firms (%)	0.571	0.568	-0.45%
growth rate by domestic incumbents (%)	0.454	0.458	0.84%
growth rate from domestic internal innovation (%)	0.438	0.443	1.06%
growth rate from domestic external innovation (%)	0.016	0.015	-5.09%
growth rate from domestic startups (%)	0.117	0.110	-5.44%

F Counterfactual: Increased Competitive Pressure by Domestic Startups

In this exercise, we reduce the cost of external R&D for potential startups $\widetilde{\chi}^e$ by 11.34%. This raises the aggregate creative destruction arrival rate $\overline{x}$ from 0.120 to 0.123 (a 2.98% increase), which is identical to the change observed in the prior exercise, where we increased the foreign creative destruction arrival rate by 20%.

Table F1 presents the results. Given the unchanged aggregate creative destruction arrival rate, all moments pertaining to individual incumbent firms closely resemble those in the main text. Nevertheless, the total mass of domestic firms, the total mass of domestic startups, and the probability of external innovation by potential startups experience an increase in this case. This shift is attributed to the heightened competitive pressure resulting from a higher mass of domestic startups, rather than foreign firms. This exercise underscores examining moments related to the number of domestic firms and startups helps in identifying the source of heightened competitive pressure (whether it emanates from the domestic entry margin or foreign firm entry).

Table F2 displays the growth rate decomposition for this economy. In contrast to the case where heightened competitive pressure is attributed to foreign firms, the contributions of domestic firms to aggregate growth are positive in this case. Nevertheless, the contribution of external innovation by domestic firms remains negative in this case as well.

Table F1: Aggregate Moment Change: Economy with Low Entry Costs

description	before	after	% change
total mass of domestic firms	0.462	0.494	6.80%
total mass of domestic startups	0.025	0.030	18.82%
R&D to sales ratio (%)	4.171	4.160	-0.24%
avg. number of products	1.767	1.706	-3.48%
avg. growth rate (%)	1.049	1.060	1.05%
prob. of external innovation, potential startups	0.033	0.040	19.57%

Table F2: Aggregate Growth Rate Decomposition with Low Entry Costs

Description	Before	After	% Change
average growth rate (g, %)	1.049	1.060	1.05%
growth rate by outside firms (%)	0.155	0.139	-10.51%
growth rate by domestic firms (%)	0.894	0.914	2.14%
growth rate by domestic incumbents (%)	0.774	0.771	-0.36%
growth rate from domestic internal innovation (%)	0.544	0.566	3.98%
growth rate from domestic external innovation (%)	0.230	0.205	-10.64%
growth rate from domestic startups (%)	0.121	0.143	18.18%

References

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