Online Appendix for "Heterogeneous Innovations and Growth under Imperfect Technology Spillovers" (NOT FOR PUBLICATION)

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A Appendix: Proofs of Propositions

A.1 Proof of Lemma 1

Consider the following two cases: 1) no ownership change between $t-1$ and t, and 2) ownership change happens between $t-1$ and t. In scenario 1), $q_{j,t} = \Delta_{j,t} q_{j,t-1}$ with only $\Delta_{j,t} \in \{\Delta^1 = 1, \Delta^2 = \lambda\}$ as a result of own-innovation. In scenario 2), $q_{j,t} = \eta q_{j,t-2}$ holds. Let's consider all possible cases where i) $\Delta_{j,t} = 1$, ii) $\Delta_{j,t} = \lambda$, iii) $\Delta_{j,t} = \eta$, iv) $\Delta_{j,t} = \frac{\eta}{\lambda}$, v) $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geq m > 0$, and vi) $\Delta_{j,t} = \frac{\lambda^n}{\eta^m}$ with $n > m > 0$. These are the only possible values Δ can assume, given that product quality can only be adjusted by three step sizes (1, λ, and η) between two periods without technology regression ($q_t < q_{t-1}$).

- i) $\Delta_{j,t} = 1$: For this to be true, $q_{j,t} = q_{j,t-1}$ should hold. Since $q_{j,t} = \eta q_{j,t-2}$, we need $q_{j,t-1} = \eta q_{j,t-2}$. This is possible if there was creative destruction between $t-2$ and $t-1$, and no own-innovation between $t-3$ and $t-1$, leading to $q_{j,t-2} = q_{j,t-3}$.

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- ii) $\Delta_{j,t} = \lambda$: For this to be true, $\Delta_{j,t-1} = \frac{\eta}{\lambda}$ should hold, as $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}} = \frac{\eta q_{j,t-2}}{\Delta_{j,t-1} q_{j,t-2}}$. This can be possible if there were own-innovation between $t-3$ and $t-2$, and creative destruction between $t-2$ and $t-1$, but no own-innovation between $t-2$ and $t-1$. In this case, $q_{j,t-2} = \lambda q_{j,t-3}$ and $q_{j,t-1} = \eta q_{j,t-3}$ holds, and thus $\Delta_{j,t-1} = \frac{q_{j,t-1}}{q_{j,t-2}} = \frac{\eta q_{j,t-3}}{\lambda q_{j,t-3}} = \frac{\eta}{\lambda}$ follows. So we have shown that both $\Delta_{j,t} = \lambda$ and $\Delta_{j,t} = \frac{\eta}{\lambda}$ are possible, and $\Delta_{j,t} = \frac{\eta}{\lambda}$ can be realized only through creative destruction between $t-1$ and t.

- iii) $\Delta_{j,t} = \eta$: For this to be true, $q_{j,t-1} = q_{j,t-2}$ should hold. This is possible if there was neither ownership change nor own-innovation between $t-1$ and $t-2$.

- iv) $\Delta_{j,t} = \frac{\eta}{\lambda}$: This follows the illustration in case ii)

- v) $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geq m > 0$: Suppose this is the case. As $\Delta_{j,t} \notin \{\Delta^1 = 1, \Delta^2 = \lambda\}$, there should be an ownership change between $t-1$ and t. Thus $q_{j,t} = \eta q_{j,t-2}$ holds, implying $q_{j,t-1} = \frac{\lambda^m}{\eta^{n-1}} q_{j,t-2}$. Note that $m \leq n-1$ is not possible without technology regression. Thus, $m = n$ (as $m > n-1$ and $n \geq m > 0$). If $\frac{\lambda^m}{\eta^{m-1}} < 1$, this implies technology regression and can be ruled out. Suppose $\frac{\lambda^m}{\eta^{m-1}} > 1$. If $m = 1$, we are back to the cases ii) and iv). Suppose $m > 1$. As $\frac{\lambda^m}{\eta^{m-1}} \neq 1$ or λ, there should be an ownership change between $t-2$ and $t-1$. Thus, $q_{j,t-1} = \eta q_{j,t-3}$ holds, implying $q_{j,t-2} = \frac{\eta^m}{\lambda^m} q_{j,t-3}$. If $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ is possible, $q_{j,t-s} = \frac{\eta^m}{\lambda^m} q_{j,t-s-1}$ holds for even numbers s, and $\frac{\lambda^m}{\eta^{m-1}} q_{j,t-s-1}$ holds for odd numbers s. Thus, in this case, either $q_{j,1} = \frac{\eta^m}{\lambda^m} q_{j,0}$ or $q_{j,1} = \frac{\lambda^m}{\eta^{m-1}} q_{j,0}$ must hold, which can be ruled out (or we assume this case does not occur). Thus, $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geq m > 0$ is not possible.

- vi) $\Delta_{j,t} = \frac{\lambda^n}{\eta^m}$ with $n > m > 0$: Following the same argument, this case is not possible.

Therefore $\Delta_{j,t}$ can assume only the four values of $\{1, \lambda, \eta, \frac{\eta}{\lambda}\}$.

A.2 Proof of Proposition 1

Using the conjectured value function, we can decompose the expected value into two parts with the linearity of expectation: the expected value of existing product lines $\mathbb{E}\big[\sum_{\ell=1}^{2} A_\ell \sum_{j \in \mathcal{J}^f | (\Delta_j' | \Delta_j) = \Delta^\ell} \Delta^\ell q_j\big]$ and the expected value for the new product line added through creative destruction $\mathbb{E}\big[\sum_{\ell=1}^{4} A_\ell \, I_{\{\eta/\Delta_j = \Delta^\ell\}} \, \frac{\eta}{\Delta_j} \, q_j\big]$. As the realization of own-innovation outcomes and the creative destruction are independent of the realization of creative destruction, the expected value of a new product line becomes:

$$\mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell \, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}} \, \frac{\eta}{\Delta_j} \, q_j\right] = \sum_{I^x=0}^{1} x^{I^x}(1-x)^{1-I^x}\mathbb{E}_{q_j,\Delta_j}\left[\sum_{\ell=1}^{4} A_\ell \, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}} \, I^x \, \frac{\eta}{\Delta_j} \, q_j\right]$$
$$= x\left[\frac{1-z^3}{2}A_1\mu(\Delta^3) + \left(1-\frac{z^4}{2}\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right]\overline{q}\,.$$

The terms in the bracket arise from the random property of creative destruction. The assigned product can have a technology gap of Δ^ℓ with a probability of $\mu(\Delta^\ell)$, and the probability of taking over this product line depends on its technology gap. Integrating over all possible qualities q_j over the entire set of available products gives us $\overline{q}$.[1]

The expected value of existing product lines can further be broken down into the four cases of Δ and integrated as $\sum_{\widetilde{\ell}=1}^{4} \mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j\in\mathcal{J}^f|(\Delta_j'|\Delta_j=\Delta^{\widetilde{\ell}})=\Delta^\ell} \Delta^\ell q_j\right]$. To simplify the derivation, we reorder product quality q_j by its technology gap Δ_j and categorize it into the following four groups: $q_1^f = \{q_{j_1}, q_{j_2}, \ldots, q_{j_{n_f^1}}\}$; $q_2^f = \{q_{j_{n_f^1+1}}, \ldots, q_{j_{n_f^1+n_f^2}}\}$; $q_3^f = \{q_{j_{n_f^1+n_f^2+1}}, \ldots, q_{j_{n_f^1+n_f^2+n_f^3}}\}$; and $q_4^f = \{q_{j_{n_f^1+n_f^2+n_f^3+1}}, \ldots, q_{j_{n_f^1+n_f^2+n_f^3+n_f^4}}\}$, $q^f = \cup_{\widetilde{\ell}=1}^{4} q_{\widetilde{\ell}}^f$.

If $\Delta = \Delta^1$ ($\widetilde{\ell}=1$), the expected value can be rephrased as $\sum_{i=1}^{n_f^1}\left[A_1(1-\overline{x})(1-z_i^1) + \lambda A_2(1-\overline{x})z_i^1\right]q_{j_i}$; if $\Delta = \Delta^2$ ($\widetilde{\ell}=2$), it becomes $\sum_{i=n_f^1+1}^{n_f^1+n_f^2}\left[A_1(1-\overline{x})(1-z_i^2) + \lambda A_2 z_i^2\right]q_{j_i}$; if $\Delta = \Delta^3$ ($\widetilde{\ell}=3$), it is $\sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4}\left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z_i^3) + \lambda A_2 z_i^3\right]q_{j_i}$; and if $\Delta = \Delta^4$ ($\widetilde{\ell}=4$), it is $\sum_{i=n_f-n_f^4}^{n_f}\left[A_1(1-\overline{x})(1-z_i^4) + \lambda A_2\big(1-\frac{1}{2}\overline{x}\big)z_i^4\right]q_{j_i}$.

The $B\overline{q}$ portion of the conjectured value function in $\mathbb{E}\left[V\left(\Phi^{f\prime} \mid \Phi^f\right)\Big|\{z_j\}_{j\in\mathcal{J}^f}, x\right]$ can be expressed as $\mathbb{E}B\overline{q}' = B(1+g)\overline{q}$, where g denotes the growth rate of product quality in a balanced growth path (BGP) equilibrium. Plugging this into the conjectured value function, we can rephrase the original value function as:

$$\sum_{i=1}^{n_f^1} A_1 q_{j_i} + \sum_{i=n_f^1+1}^{n_f^1+n_f^2} A_2 q_{j_i} + \sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4} A_3 q_{j_i} + \sum_{i=n_f-n_f^4+1}^{n_f} A_4 q_{j_i} + B\overline{q} =$$

[1] Note that individual firms only have information about the distribution of technology gaps $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ and the average quality level $\overline{q}$. That is, for an individual firm, a technology gap and product quality are independent considerations.

$$\max_{\substack{x\in[0,\bar{x}],\\ \{z_i\in[0,\bar{z}]\}_{i=1}^{n_f}}} \left\{ \begin{array}{l} \sum_{i=1}^{n_f}\left[\pi q_{j_i} - \hat{\chi} z_i^{\hat{\psi}} q_{j_i}\right] - \bar{q}\widetilde{\chi} x^{\widetilde{\psi}} \\ +\widetilde{\beta}\sum_{i=1}^{n_f^1}\left[A_1(1-\overline{x})(1-z_i^1) + \lambda A_2(1-\overline{x})z_i^1\right] q_{j_i} \\ +\widetilde{\beta}\sum_{i=n_f^1+1}^{n_f^1+n_f^2}\left[A_1(1-\overline{x})(1-z_i^2) + \lambda A_2 z_i^2\right] q_{j_i} \\ +\widetilde{\beta}\sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4}\left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z_i^3) + \lambda A_2 z_i^3\right] q_{j_i} \\ +\widetilde{\beta}\sum_{i=n_f-n_f^4}^{n_f}\left[A_1(1-\overline{x})(1-z_i^4) + \lambda A_2\left(1-\frac{1}{2}\overline{x}\right)z_i^4\right] q_{j_i} \\ +\widetilde{\beta}x\left[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4)\right. \\ \qquad\left. +A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right]\overline{q} \\ +\widetilde{\beta}B(1+g)\overline{q} \end{array} \right\}.$$

By taking the first-order conditions with respect to each innovation intensity, we get the optimal innovation decision rules, which depend solely on technology gaps. Substituting these optimal innovation intensities into the value function, equating the left-hand side (LHS) to the right-hand side (RHS), and collecting terms, we obtain the five coefficients of the conjectured value function.

A.3 Proof of Corollary 1

Define $\widetilde{z}^\ell = \frac{\widehat{\psi}\widehat{\chi}}{\widetilde{\beta}}\left(z^\ell\right)^{(\widehat{\psi}-1)}$. Then $z^\ell > z^{\ell'} \Leftrightarrow \widetilde{z}^\ell > \widetilde{z}^{\ell'}$ for all $\ell, \ell' \in [1,4]\cap\mathbb{Z}$ under the condition $\widehat{\psi} > 1$. Given $\widetilde{z}^2 - \widetilde{z}^3 = \frac{1}{2}\overline{x}A_1 > 0$, $\widetilde{z}^2 - \widetilde{z}^1 = \overline{x}\lambda A_2 > 0$, $\widetilde{z}^2 - \widetilde{z}^4 = \frac{1}{2}\overline{x}\lambda A_2 > 0$, and $\widetilde{z}^4 - \widetilde{z}^1 = \frac{1}{2}\overline{x}\lambda A_2 > 0$, we can obtain the following relationships: $z^2 > z^3$, $z^2 > z^1$, $z^2 > z^4$, and $z^4 > z^1$. Given $\widetilde{z}^1 = (1-\overline{x})\left[\lambda A_2 - A_1\right] > 0$ in equilibrium, $\lambda A_2 - A_1 > 0$ holds, and $\widetilde{z}^3 > \widetilde{z}^4 \Leftrightarrow z^3 > z^4$ is derived. Thus, the order of $\{z^\ell\}_{\ell=1}^4$ in equilibrium is $z^2 > z^3 > z^4 > z^1$.

A.4 Proof of Corollary 2

The partial derivatives of $\{z^\ell\}_{\ell=1}^4$ with respect to $\overline{x}$ are (after removing the common terms) $\left.\frac{\partial z^1}{\partial \overline{x}}\right|_{A_1,A_2}$: $-(z^1)^{2-\widehat{\psi}}\left[\lambda A_2 - A_1\right] < 0$; $\left.\frac{\partial z^2}{\partial \overline{x}}\right|_{A_1,A_2}$: $(z^2)^{2-\widehat{\psi}}A_1 > 0$; $\left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2}$: $(z^3)^{2-\widehat{\psi}}\frac{1}{2}A_1 > 0$; and $\left.\frac{\partial z^4}{\partial \overline{x}}\right|_{A_1,A_2}$: $-(z^4)^{2-\widehat{\psi}}\left[\frac{1}{2}\lambda A_2 - A_1\right] \gtrless 0$, with A_1 and A_2 fixed. As $\lambda A_2 - A_1 > 0$, it follows that $\left.\frac{\partial z^1}{\partial \overline{x}}\right|_{A_1,A_2} < 0$.

Similarly, $\frac{\partial z^2}{\partial \overline{x}}\Big|_{A_1,A_2} > \frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2}$ holds with $z^2 > z^3$, and $\frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2} > \frac{\partial z^4}{\partial \overline{x}}\Big|_{A_1,A_2}$ holds with $z^3 > z^4$ and $\lambda A_2 - A_1 > 0$. However, the sign for $\frac{1}{2}\lambda A_2 - A_1$ remains ambiguous.

A.5 Proof of Potential Startups' Problem

With the value function defined for incumbents, we have $\mathbb{E}V(\{(q_j', \Delta_j')\}) = x_e\big[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \big(1-\frac{1}{2}z^4\big)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\big]\overline{q} + x_e\big[\frac{1}{2}(1-z^3)\mu(\Delta^3) + \big(1-\frac{1}{2}z^4\big)\mu(\Delta^4) + \mu(\Delta^1) + (1-z^2)\mu(\Delta^2)\big]B(1+g)\overline{q}$, from which the optimal creative destruction choice for potential startups can be derived.

A.6 Proof of Proposition 2

In this model, the output growth rate is the same as the product quality growth rate. For product j with quality q_j and a technology gap of $\Delta_j = \Delta^\ell$, we can derive the following law of motion of q_j:

$$
\begin{array}{llll|llll}
\Delta^1: & q_j' = \Delta^1 q_j & \text{prob.} & (1-\overline{x})(1-z^1) & \Delta^2: & q_j' = \Delta^1 q_j & \text{prob.} & (1-\overline{x})(1-z^2) \\
 & q_j' = \Delta^2 q_j & \text{prob.} & (1-\overline{x})z^1 & & q_j' = \Delta^2 q_j & \text{prob.} & z^2 \\
 & q_j' = \Delta^3 q_j & \text{prob.} & \overline{x} & & q_j' = \Delta^3 q_j & \text{prob.} & 0 \\
 & q_j' = \Delta^4 q_j & \text{prob.} & 0 & & q_j' = \Delta^4 q_j & \text{prob.} & \overline{x}(1-z^2) \\
\Delta^3: & q_j' = \Delta^1 q_j & \text{prob.} & 1-z^3 & \Delta^4: & q_j' = \Delta^1 q_j & \text{prob.} & (1-\overline{x})(1-z^4) \\
 & q_j' = \Delta^2 q_j & \text{prob.} & z^3 & & q_j' = \Delta^2 q_j & \text{prob.} & z^4 + \overline{x}(1-z^4) \\
 & q_j' = \Delta^3 q_j & \text{prob.} & 0 & & q_j' = \Delta^3 q_j & \text{prob.} & 0 \\
 & q_j' = \Delta^4 q_j & \text{prob.} & 0 & & q_j' = \Delta^4 q_j & \text{prob.} & 0.
\end{array}
$$

Following this, we can compute the expected growth rate of q_j ($\mathbb{E}\big[q_j' \mid q_j\big]/q_j - 1$) and the aggregate growth rate in (24) by taking the expectation across all product lines.

Using the share of products owned by domestic incumbents ($s_d = \mathcal{F}_d/\mathcal{F}$), the definition of $\overline{x}$, and the evolution of product quality, the growth rate can be decomposed as follows:

$$
g = \underbrace{\big(\Delta^2 - 1\big)\, s_d \left[(1-\overline{x})z^1\mu(\Delta^1) + z^2\mu(\Delta^2) + z^3\mu(\Delta^3) + \left(1 - \frac{1}{2}\overline{x}\right) z^4\mu(\Delta^4)\right]}_{\text{own-innovation by domestic incumbent firms}}
$$

$$+ \underbrace{\left(\Delta^2-1\right)(1-s_d)\left[(1-\overline{x})z^1\mu(\Delta^1)+z^2\mu(\Delta^2)+z^3\mu(\Delta^3)+\left(1-\frac{1}{2}\overline{x}\right)z^4\mu(\Delta^4)\right]}_{\text{own-innovation by foreign firms}}$$

$$+ \underbrace{\left(\overline{\Delta^{ex}}-1\right)\mathcal{F}_d x\mu(\overline{\Delta^{ex}})}_{\text{creative destr. by domestic incumbents}} + \underbrace{\left(\overline{\Delta^{ex}}-1\right)\mathcal{E}_d x_e\mu(\overline{\Delta^{ex}})}_{\text{creative destr. by domestic startups}} + \underbrace{\left(\overline{\Delta^{ex}}-1\right)\overline{x}_o\mu(\overline{\Delta^{ex}})}_{\text{creative destr. by foreign firms}},$$

where $\overline{\Delta^{ex}} \equiv \frac{\Delta^3\mu(\Delta^1)+\Delta^4(1-z^2)\mu(\Delta^2)+\frac{1}{2}(1-z^3)\mu(\Delta^3)+\Delta^2\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)}{\mu(\Delta^1)+(1-z^2)\mu(\Delta^2)+\frac{1}{2}(1-z^3)\mu(\Delta^3)+\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)}$ is an increase in the average product quality due to creative destruction and successful business takeover, and $\mu(\overline{\Delta^{ex}}) \equiv \mu(\Delta^1)+(1-z^2)\mu(\Delta^2)+\frac{1}{2}(1-z^3)\mu(\Delta^3)+\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)$ is the share of product lines affected by creative destruction.

B Baseline Model

B.1 Illustration of Firm Innovation Decisions

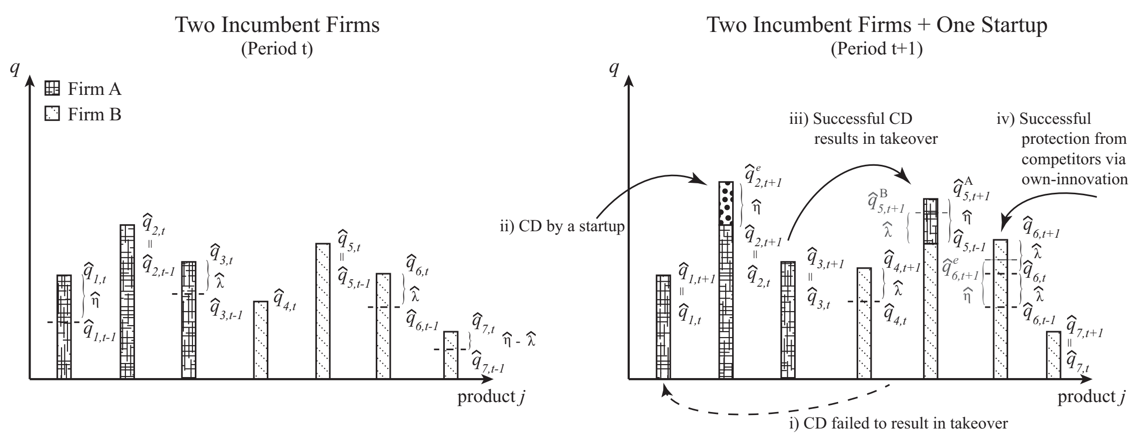

Figure B.1: Firms' Innovation and Product Quality Evolution Example

Figure B.1 illustrates the following set of examples of firm innovation decisions.[2] Suppose firm A owns products 1,2,3, and firm B owns products 4,5,6,7.

• i) Failed product takeover with coin-tossing (product 1): firm A without successful own-innovation (at t) gets $q^A_{1,t+1} = \eta q_{1,t-1}$, while firm B with successful creative destruction (CD) (at t) obtains $q^B_{1,t+1} = \eta q_{1,t-1}$. A coin is tossed, and firm A keeps the product.

[2]The bar indicates log product quality $\widehat{q}_{j,t} \equiv \log(q_{j,t})$ with $\widehat{\eta} \equiv \log(\eta)$.

• ii) Successful product takeover w/o technology gap (product 2): A potential startup with successful creative destruction (at t) can take over the market from firm A with no successful own-innovations (at both $t-1$ and t) as $q^e_{2,t+1} = \eta q_{2,t-1} > q^A_{2,t+1} = q_{2,t-1}$

• iii) Failed market protection w/o technological gap (product 5): firm A can take it over through successful creative destruction, despite concurrently successful own-innovation by firm B as $\eta q_{5,t-1} > \lambda q_{5,t-1}$

• iv) Successful market protection with a technology gap (product 6): firm B obtains $q^B_{6,t+1} = \lambda^2 q_{6,t-1}$ with consecutively successful own-innovations from $t-1$. Rivals can only innovate up to $q^e_{6,t+1} = \eta q_{6,t-1}$, which makes firm B successfully protect the market.

B.2 Product Quality Evolution

Outsider Firms Let z^ℓ_j denote the own-innovation intensity for product line j and Δ^ℓ_j denote its technology gap. Since outside firms can only learn the lagged level of technology $q_{j,-1} = q_j/\Delta^\ell_j$, the evolution of product quality in $t+1$ occurs probabilistically as follows: for $\Delta_j = \Delta^1$, q'_j is equal to $\lambda q_{j,-1}$ with prob. $(1-\overline{x})z^1_j$, $q_{j,-1}$ with prob. $(1-\overline{x})(1-z^1_j)$, and $\eta q_{j,-1}$ with prob. $\overline{x}$; for Δ^2, q'_j is equal to $\lambda^2 q_{j,-1}$ with prob. z^2_j, $\lambda q_{j,-1}$ with prob. $(1-\overline{x})(1-z^2_j)$, and $\eta q_{j,-1}$ with prob. $\overline{x}(1-z^2_j)$; for Δ^3, q'_j is equal to $\lambda\eta q_{j,-1}$ with prob. z^3_j, $\eta q_{j,-1}$ with prob. $(1-\overline{x})(1-z^3_j) + \frac{1}{2}\overline{x}(1-z^3_j)$, and $\eta q_{j,-1}$ with prob. $\frac{1}{2}\overline{x}(1-z^3_j)$; and for Δ^4, q'_j is equal to $\lambda\frac{\eta}{\lambda}q_{j,-1}$ with prob. $(1-\overline{x})z^4_j + \frac{1}{2}\overline{x}z^4_j$, $\frac{\eta}{\lambda}q_{j,-1}$ with prob. $(1-\overline{x})(1-z^4_j)$, and $\eta q_{j,-1}$ with prob. $\overline{x}(1-z^4_j) + \frac{1}{2}\overline{x}z^4_j$.

B.3 Value Function and Optimal Innovation Decisions

The conditional expectation in the value function considers the success/failure of own-innovation and creative destruction, the arrival of the creative destruction shock, outcomes of coin-tosses (c-t), the distribution of current period product quality q, and the distribution of the current period technology gap Δ^ℓ. Thus, $\mathbb{E}\big[V\big(\Phi^{f\prime}\big|\Phi^f\big)\big|\{z_j\}_{j\in\mathcal{J}^f}, x\big] = \sum_{I^{\overline{x}}_1,\dots,I^{\overline{x}}_{n_f}=0}^{1}\sum_{I^z_1,\dots,I^z_{n_f}=0}^{1}\sum_{\text{c-t}_1,\dots,\text{c-t}_{n_f}=\text{win}}^{\text{lose}}\sum_{I^x=0}^{1}\Big[\prod_{i=1}^{n_f}\overline{x}^{I^{\overline{x}}_i}(1-\overline{x})^{1-I^{\overline{x}}_i}z_i^{I^z_i}(1-z_i)^{1-I^z_i}\Big]\times\Big[x^{I^x}(1-x)^{1-I^x}\Big]\big(\tfrac{1}{2}\big)^{n_f}\mathbb{E}_{q,\Delta}V\Big(\Big[\bigcup_{i=1}^{n_f}\Big[\Big\{(\Delta'_{j_i}q_{j_i}, \Delta'_{j_i})\Big|(q_{j_i}, \Delta_{j_i}), I^{\overline{x}}_i, I^z_i, \text{c-t}_i\Big\}\setminus\{\mathbf{0}\}\Big]\Big]\bigcup\Big[\Big\{\Big(\frac{\eta}{\Delta_{-j}}I^x q_{-j}, \frac{\eta}{\Delta_{-j}}I^x\Big)\Big\}\setminus\{\mathbf{0}\}\Big]\Big)$. Note that the first term in the value function (before $\bigcup$) is the subsets of possible realizations for $\Phi^{f\prime}$ from own-innovation, creative destruction, and coin-toss. The second term in the value function (after $\bigcup$) shows the subsets of possible realizations for $\Phi^{f\prime}$ from creative destruction, where $\{q'_{j_i}\} = \{\Delta'_{j_i} q_{j_i}\} \setminus \{0\}$, and $\{q'_{-j}\} = \{\frac{\eta}{\Delta_{-j}} I^x q_{-j}\} \setminus \{0\}$. If $\Delta'_{j_i} = 0$, then firm f loses product line j_i and $\{(q'_{j_i}, \Delta'_{j_i})\} \setminus \{\mathbf{0}\} = \{\mathbf{0}\} \setminus \{\mathbf{0}\} = \emptyset$.

B.4 Technology Gap Portfolio Composition Distribution Transition

The range of $\widetilde{k}^1$ can be determined as follows: i) for $0 \leq \widetilde{k} \leq \min\{n_f - k, k\}$, the two combinations preceding the term in brackets are well defined for any $\widetilde{k}^1 \in [0, \widetilde{k}] \cap \mathbb{Z}$ and describe all possible cases; ii) if $n_f - k \geq k$, then $\tilde{k} > k$, $0 \leq \widetilde{k} - \widetilde{k}^1$, and $0 \leq \widetilde{k}^1 \leq n_f - k$ is satisfied. This gives $\widetilde{k} - k \leq \widetilde{k}^1 \leq \widetilde{k}$; and iii) if $k \geq n_f - k$, then $\widetilde{k} > n_f - k$, $0 \leq \widetilde{k} - \widetilde{k}^1$, and $0 \leq \widetilde{k}^1 \leq n_f - k$ is satisfied. Thus, $\max\{0, \widetilde{k} - k\} \leq \widetilde{k}^1 \leq n_f - k$.

By using $\widetilde{\mathbb{P}}(n_f, \widetilde{k}|n_f, k)$, the probability of $\mathcal{N} = \widetilde{\mathcal{N}}(n_f, k)$ transitioning to $\mathcal{N}' = \widetilde{\mathcal{N}}(n_f - h, \widetilde{k})$ for any $h \geq 0$ without considering creative destruction can be defined as follows: Take out h^1 numbers of product lines with $\Delta = \Delta^1$, and $h - h^1$ numbers of product lines with $\Delta = \Delta^2$ from $\widetilde{\mathcal{N}}(n_f, k)$, then compute the probability of $\widetilde{\mathcal{N}}(n_f - h, k - (h - h^1))$ transitioning to $\widetilde{\mathcal{N}}(n_f - h, \widetilde{k})$ with $\widetilde{\mathbb{P}}\big(n_f - h, \widetilde{k}|n_f - h, k - (h - h^1)\big)$ for all feasible h^1. Then, for $0 \leq h < n_f$, $n_f \geq 1$, $0 \leq \widetilde{k} \leq n_f - h$, and $0 \leq k \leq n_f$, $\widetilde{\mathbb{P}}\big(n_f - h, \widetilde{k}|n_f, k\big) = \sum_{h^1=\max\{0,h-k\}}^{\min\{h, n_f-k\}} \Big[\begin{pmatrix} n_f - k \\ h^1 \end{pmatrix}\begin{pmatrix} k \\ h - h^1 \end{pmatrix}\overline{x}^h(1 - z^2)^{h-h^1}\widetilde{\mathbb{P}}\big(n_f - h, \widetilde{k}|n_f - h, k - (h - h^1)\big)\Big]$; for $h = n_f \geq 1$, $\widetilde{k} = 0$, and $0 \leq k \leq n_f$, $\widetilde{\mathbb{P}}\big(n_f - h, \widetilde{k}|n_f, k\big) = \overline{x}^{n_f}(1 - z^2)^k$; and 0 otherwise. The range for h^1 is defined from above, ensuring $0 \leq h - h^1 \leq k$ and $0 \leq h^1 \leq n_f - k$ for any h^1.

With $\widetilde{\mathbb{P}}\big(n_f - h, \widetilde{k}|n_f, k\big)$, other possible cases can be described for each case. For example, the probability of $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ to $\mathcal{N}' = (n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0)$ for $h \geq -1$ is defined as $\mathbb{P}\big(n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0\,\big|n_f, n_f - k, k, 0, 0\big) = \widetilde{\mathbb{P}}\big(n_f - h, \widetilde{k}\big|n_f, k\big)(1 - x\overline{x}_{\text{takeover}}) + \widetilde{\mathbb{P}}\big(n_f - h - 1, \widetilde{k}\big|n_f, k\big)\mu(\Delta^3)\frac{1}{2}x(1 - z^3) + \widetilde{\mathbb{P}}\big(n_f - h - 1, \widetilde{k} - 1\big|n_f, k\big)\mu(\Delta^4)x\left(1 - \frac{1}{2}z^4\right)$. The first term is the probability of $\mathcal{N}$ transitioning to $\mathcal{N}'$ directly via the change in the firm's existing technology gap portfolio composition with unsuccessful creative destruction. The second term is the probability of $\mathcal{N}$ to $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k})$, where successful creative destruction adds one product line with $\Delta' = \Delta^1$. Since the next period technology gap of product line j from successful creative destruction is $\Delta'_j = \frac{q'_j}{q_j} = \frac{\eta q_{j,-1}}{\Delta_j q_{j,-1}} = \frac{\eta}{\Delta_j}$, firm needs to take over a product line with a technology gap of $\Delta = \Delta^3 = 1 + \eta$ to have a product line with a technology gap of Δ^1 in the next period. The third term is the probability of $\mathcal{N}$ to $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k} - 1)$, where successful creative destruction adds one product line with $\Delta' = \Delta^2$ by taking over a product line with a technology gap of $\Delta = \Delta^4$. For $h = -1$, the first term becomes zero by the definition of $\widetilde{\mathbb{P}}(\cdot|\cdot)$. Thus this probability is well defined for any $h \geq -1$.

With the computed probabilities of transitions between technology gap portfolio compositions, we can now define the inflows and outflows of a given technology gap portfolio. Let $\mathcal{F}$ denote the total mass of firms in the economy and $\mu(\mathcal{N})$ represent the share of firms with technology gap portfolio $\mathcal{N}$. Thus, $\widetilde{\mu}(\mathcal{N}) = \mathcal{F}\mu(\mathcal{N})$. Then, for example, inflows and outflows for $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ can be described as follows: for $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ with $n_f \geq 2$, any firm whose next period technology gap portfolio is not $\mathcal{N}$ is counted as outflows, followed by $\mathit{outflow}(n_f, n_f - k, k, 0, 0) = [1 - \mathbb{P}(n_f, n_f - k, k, 0, 0 \big| n_f, n_f - k, k, 0, 0)] \times \mathcal{F}\mu(n_f, n_f - k, k, 0, 0)$. Any firm with a total number of product lines $n \geq n_f - 1$ can have a technology gap portfolio composition equal to $\mathcal{N}$ through the combinations of own-innovation and creative destructions. Thus, for the maximum number of product lines $\bar{n}_f$, $\mathit{inflow}(n_f, n_f - k, k, 0, 0) = \mathcal{F}\sum_{n=n_f-1}^{\overline{n}_f}\sum_{\widetilde{k}=0}^{n}\big[\mu(n, n - \widetilde{k}, \widetilde{k}, 0, 0)\mathbb{P}(n_f, n_f - k, k, 0, 0 \big| n, n - \widetilde{k}, \widetilde{k}, 0, 0) + \mu(n, n - 1 - \widetilde{k}I_{\{n>1\}}, \widetilde{k}I_{\{n>1\}}, 1, 0)\mathbb{P}(n_f, n_f - k, k, 0, 0 \big| n, n - 1 - \widetilde{k}I_{\{n>1\}}, \widetilde{k}I_{\{n>1\}}, 1, 0) + \mu(n, n - 1 - \widetilde{k}I_{\{n>1\}}, \widetilde{k}I_{\{n>1\}}, 0, 1)\mathbb{P}(n_f, n_f - k, k, 0, 0 \big| n, n - 1 - \widetilde{k}I_{\{n>1\}}, \widetilde{k}I_{\{n>1\}}, 0, 1)\big] - \mathcal{F}\mu(n_f, n_f - k, k, 0, 0)\mathbb{P}(n_f, n_f - k, k, 0, 0 \big| n_f, n_f - k, k, 0, 0)$.[3]

C Simple Three-Period Heterogeneous Innovation Model

To analyze firms' innovation incentives and derive testable predictions, we examine a three-period economy with two product markets and three firms. In period 0, the economy starts with two product markets, market 1 and 2, with initial market-specific technologies $q_{1,0}$, and $q_{2,0}$, respectively. There are two firms in play, firm A and B. Firm A starts with product market 1 and an initial own-innovation probability $z_{1,0}$. Firm B, on the other hand, starts only with an initial creative destruction probability $x_{2,0}$, and can operate and produce in period 1 (but not in period 0). If creative destruction fails, firm B still keeps market 2 but produces with initial quality $q_{2,0}$. Thus, at the beginning of period 1, product qualities in the two markets are $q_{1,1} = \lambda q_{1,0}$ with probability $z_{1,0}$, $q_{1,1} = q_{1,0}$ with probability $1 - z_{1,0}$, $q_{2,1} = \eta q_{2,0}$ with probability $x_{2,0}$, and $q_{2,0}$ with probability

[3] Descriptions for other cases are available upon request.

$1 - x_{2,0}$, where $\lambda^2 > \eta > \lambda > 1$ represent innovation step sizes.

In period 1, the focal period, an outside firm engages in creative destruction to potentially take over the two product markets in period 2. The success of the outside firm in creative destruction is determined by the probability x_1^e for each product market. Additionally, there is a news shock in period 1 concerning the profit for period 2, possibly including an increase in foreign demand. Subsequently, the two incumbents utilize their given technologies to produce and invest in own-innovation and creative destructions. At the beginning of period 2, all innovation outcomes are realized, and then technological competition in each product market takes place. Only the firm with the highest technology in each product market continues producing. The economy ends after period 2.

In period 1, incumbent firm $i \in \{A, B\}$ invests $R_{j,1}^{\text{in}}$ in own-innovation for $j \in \{1, 2\}$, achieving a success probability of $z_{j,1}$. The R&D production function is $z_{j,1} = \left(R_{j,1}^{\text{in}}/\widehat{\chi}q_{j,1}\right)^{0.5}$. Successful own-innovation increases next-period product quality by $\lambda > 1$. Thus, the period 2 product quality for firm i becomes $q_{j,2}^i = \lambda q_{j,1}$ with prob. $z_{j,1}$, and $q_{j,2}^i = q_{j,1}$ with prob. $1 - z_{j,1}$. Similarly, firm i invests $R_{-j,1}^{\text{ex}}$ to learn the period 0 technology used by firm $-i \neq i$ and improve it, which determines the success probability of creative destruction $x_{-j,1}$. The R&D production function is $x_{-j,1} = \left(R_{-j,1}^{\text{ex}}/\widetilde{\chi}q_{-j,0}\right)^{0.5}$, where $-j$ is owned by $-i$. Successful creative destruction enhances product quality relative to the lagged-period quality by $\eta > 1$. Thus, product $-j$'s quality in period 2 for firm i is $q_{-j,2}^i = \eta q_{-j,0}$ with prob. $x_{-j,1}$, and $q_{-j,2}^i = \emptyset$ with prob. $1 - x_{-j,1}$, where the symbol $\emptyset$ means firm i failed to acquire the production technology for product $-j$.

Optimal Innovation Decisions and Theoretical Predictions Assume that in a given product market j and period t, firms receive an instantaneous profit of $\pi_{j,t}q_{j,t}$ where $q_{j,t}$ is the product quality and $\pi_{j,t}$ is a market-period-specific constant known to firms before each period begins. Because there are only two products, incumbents and the outside firm can perform creative destruction on the same product. For simplicity in the model, we further assume that the outside firm can engage in creative destruction only if an incumbent fails to do so, following Garcia-Macia et al. (2019). Then the profit maximization problem for firm i in product market j with quality $q_{j,1}$ in period 1 can be written as $V(q_{j,1}) = \max_{\{z_{j,1},x_{-j,1}\}} \{\pi_{j,1}q_{j,1} - \widehat{\chi}(z_{j,1})^2 q_{j,1} - \widetilde{\chi}(x_{-j,1})^2 q_{-j,0} + (1 - x_{j,1})(1 - x_1^e)\left[(1 - z_{j,1})\pi_{j,2}q_{j,1} + z_{j,1}\pi_{j,2}\lambda q_{j,1}\right] + (x_{j,1} + (1 - x_{j,1})x_1^e)\left[z_{j,1}\pi_{j,2}\lambda q_{j,1}\mathcal{I}_{\{\lambda q_{j,1} > \eta q_{j,0}\}} + \frac{1}{2}(1 -\right.$

$z_{j,1})\pi_{j,2}q_{j,1}\mathcal{I}_{\{q_{j,1}=\eta q_{j,0}\}}\big]+x_{-j,1}\big[(1-z_{-j,1})\pi_{-j,2}\eta q_{-j,0}\mathcal{I}_{\{\eta q_{-j,0}>q_{-j,1}\}}+z_{-j,1}\pi_{-j,2}\eta q_{-j,0}\mathcal{I}_{\{\eta q_{-j,0}>\lambda q_{-j,1}\}}$ $+\frac{1}{2}(1-z_{-j,1})\pi_{-j,2}\eta q_{-j,0}\mathcal{I}_{\{\eta q_{-j,0}=q_{-j,1}\}}+\frac{1}{2}z_{-j,1}\pi_{-j,2}\eta q_{-j,0}\mathcal{I}_{\{\eta q_{-j,0}=\lambda q_{-j,1}\}}\big]\big\}$, where $\mathcal{I}_{\{\cdot\}}$ is an indicator function that captures the possible relationships between the technologies of the three firms in period 2 within a given market. The first three terms show the period 1 profit net of total R&D cost.

The first bracket represents the incumbent's expected profit from market j if neither the incumbent nor the outside firm succeeds in creative destruction in market j. The second bracket represents the expected profit from market j if either the other incumbent or the outside firm succeeds in creative destruction in market j. The third bracket represents the expected profit from market $-j$ if firm i succeeds in creative destruction in market $-j$. The terms following $\frac{1}{2}$ account for scenarios where two firms could potentially produce the same quality product, triggering a coin-toss tiebreaker rule.

The interior solutions to this problem are: for $q_{j,1}=q_{j,0}$, $z^*_{j,1}=\frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda-1)(1-x^*_{j,1})(1-x^e_1)$; for $q_{j,1}=\lambda q_{j,0}$, $z^*_{j,1}=\frac{\pi_{j,2}}{2\widehat{\chi}}\big[\lambda-(1-x^*_{j,1})(1-x^e_1)\big]$; for $q_{j,1}=\eta q_{j,0}$, $z^*_{j,1}=\frac{\pi_{j,2}}{2\widehat{\chi}}\big[\lambda-\frac{1}{2}-\frac{1}{2}(1-x^*_{j,1})(1-x^e_1)\big]$; for $q_{-j,1}=q_{-j,0}$, $x^*_{-j,1}=\frac{\eta\pi_{-j,2}}{2\widetilde{\chi}}$; for $q_{-j,1}=\lambda q_{-j,0}$, $x^*_{-j,1}=\frac{\eta\pi_{-j,2}}{2\widetilde{\chi}}(1-z^*_{-j,1})$; and for $q_{-j,1}=\eta q_{-j,0}$, $x^*_{-j,1}=\frac{\eta\pi_{-j,2}}{2\widetilde{\chi}}\frac{1}{2}(1-z^*_{-j,1})$, which maximize the firm profit considering the technology gap of its own and others, as well as the potential outcomes of own-innovation and creative destruction by all firms involved.

Proposition C.1. *For each $q_{j,1}$ and for $\lambda^2>\eta>\lambda>1$, we can order own-innovation intensities as $z^*_{j,1}\big|_{q_{j,1}=\lambda q_{j,0}}>z^*_{1,1}\big|_{q_{j,1}=\eta q_{j,0}}>z^*_{j,1}\big|_{q_{j,1}=q_{j,0}}$. Furthermore, $\frac{\partial z^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=\lambda q_{j,0}}>\frac{\partial z^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=\eta q_{j,0}}>0>\frac{\partial z^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=q_{j,0}}$.*

Proof. The first part is straightforward with simple algebra. The second part is proved as follows. For $q_{j,1}=q_{j,0}$, we have $\frac{\partial z_{j,1}}{\partial x^e_1}=-\frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda-1)\big[(1-x_{j,1})+(1-x^e_1)\frac{\partial x_{j,1}}{\partial x^e_1}\big]$, and $\frac{\partial x_{j,1}}{\partial x^e_1}=0$. Thus, the following is obtained: $\frac{\partial z_{j,1}}{\partial x^e_1}=-\frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda-1)(1-x_{j,1})<0$. For $q_{j,1}=\lambda q_{j,0}$, we have $\frac{\partial z_{j,1}}{\partial x^e_1}=\frac{\pi_{j,2}}{2\widehat{\chi}}\big[1-x_{j,1}+(1-x^e_1)\frac{\partial x_{j,1}}{\partial x^e_1}\big]$ and $\frac{\partial x_{j,1}}{\partial x^e_1}=-\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x^e_1}$. Thus, the following holds: $\frac{\partial z_{j,1}}{\partial x^e_1}=(1-x_{j,1})\big[\frac{2\widehat{\chi}}{\pi_{j,2}}+\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}(1-x^e_1)\big]^{-1}>0$, hence $\frac{\partial x_{j,1}}{\partial x^e_1}=-\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x^e_1}<0$. For $q_{j,1}=\eta q_{j,0}$, we have $\frac{\partial z_{j,1}}{\partial x^e_1}=\frac{\pi_{j,2}}{2\widehat{\chi}}\frac{1}{2}\big[1-x_{j,1}+(1-x^e_1)\frac{\partial x_{j,1}}{\partial x^e_1}\big]$, and $\frac{\partial x_{j,1}}{\partial x^e_1}=-\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{1}{2}\frac{\partial z_{j,1}}{\partial x^e_1}$. This gives $\frac{\partial z_{j,1}}{\partial x^e_1}=(1-x_{j,1})\big[\frac{4\widehat{\chi}}{\pi_{j,2}}+\frac{\eta\pi_{j,2}}{4\widetilde{\chi}}(1-x^e_1)\big]^{-1}>0$, hence $\frac{\partial x_{j,1}}{\partial x^e_1}=-\frac{1}{2}\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x^e_1}<0$ holds. With $x^*_{j,1}$ and $\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\in(0,1)$, along with the restriction $4\widehat{\chi}>\pi_{j,2}$, the following holds: $\frac{4\widehat{\chi}}{\pi_{j,2}}+\frac{\eta\pi_{j,2}}{4\widetilde{\chi}}(1-x^e_1)>\frac{2\widehat{\chi}}{\pi_{j,2}}+\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}(1-x^e_1)$. Therefore, we get $\frac{\partial z^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=\lambda q_{j,0}}>\frac{\partial z^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=\eta q_{j,0}}$ $\square$

The second part of proposition C.1 suggests that firms without a technology gap decrease their own-innovation when facing a higher probability of creative destruction in their own markets. This is because they cannot enhance their product protection through own-innovation. Conversely, firms with a significant technological advantage do not substantially increase their own-innovation in response to creative destruction from outsiders, as the risk of losing their own product market is minimal. In intermediate cases, firms intensify their own-innovation response to creative destruction from outsiders to reduce the probability of losing their market.

Higher innovation in period 0 increases the probability of achieving a high technology gap in period 1, thereby aiding firms in market protection. To understand how past innovation intensity influences the firm's current decision on own-innovation when facing a higher probability of encountering a competitor, x_1^e, we define the expected value of own-innovation intensity in period 1 as $\bar{z}_1^* = z_{1,1}^*\big|_{q_{1,1}=q_{1,0}} \frac{1}{2}(1-z_{1,0}) + z_{2,1}^*\big|_{q_{2,1}=q_{2,0}} \frac{1}{2}(1-x_{2,0}) + z_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} \frac{1}{2} z_{1,0} + z_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} \frac{1}{2} x_{2,0}$, where $\frac{1}{2}$ accounts for the two products. Proposition C.1 provides the following result:

Corollary C.1 (Market-Protection Effect)**.** *The impact of period 0 innovation intensities, $z_{1,0}$ and $x_{2,0}$, on expected own-innovation in period 1 can be characterized as follows: $\frac{\partial \bar{z}_1^*}{\partial x_1^e \partial z_{1,0}} > 0$, and $\frac{\partial \bar{z}_1^*}{\partial x_1^e \partial x_{2,0}} > 0$.*

Proof. From $\bar{z}_1^*$, we know that $\frac{\partial \bar{z}_1^*}{\partial z_{1,0}} = \frac{1}{2}\left(z_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} - z_{1,1}^*\big|_{q_{1,1}=q_{1,0}}\right) > 0$ and $\frac{\partial \bar{z}_1^*}{\partial x_{2,0}} = \frac{1}{2}\left(z_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} - z_{2,1}^*\big|_{q_{2,1}=q_{2,0}}\right) > 0$, where the signs can be derived from proposition C.1. The results follow from proposition C.1. □

Corollary C.1 suggests that intensive innovation in the previous period prompts firms to increase own-innovation in response to higher competitive pressure. As indicated by the optimal decision rule, firms' decisions regarding creative destruction also depend on the past innovation decisions of other firms, which is outlined in the following proposition.

Proposition C.2. *For each $q_{j,1}$ and for $\lambda^2 > \eta > \lambda > 1$, we can order creative destruction intensities as follows: $x_{j,1}^*\big|_{q_{j,1}=q_{j,0}} > x_{j,1}^*\big|_{q_{j,1}=\lambda q_{j,0}} > x_{j,1}^*\big|_{q_{j,1}=\eta q_{j,0}}$. Furthermore, $\frac{\partial x_{j,1}^*}{\partial x_1^e}\big|_{q_{j,1}=q_{j,0}} = 0$, $\frac{\partial x_{j,1}^*}{\partial x_1^e}\big|_{q_{j,1}=\lambda q_{j,0}} < 0$, and $\frac{\partial x_{j,1}^*}{\partial x_1^e}\big|_{q_{j,1}=\eta q_{j,0}} < 0$. Proof: See the proof for Proposition C.1*

Proposition C.2 implies that firms decrease creative destruction if incumbents hold a higher technology advantage, as it becomes more difficult to displace them in the market through creative destruction. In markets where there is a technological barrier (technology gap > 1), firms also reduce their creative destruction in response to increased creative destruction by outside firms. This is because incumbents in these markets respond defensively by increasing own-innovation (proposition C.1). To understand how the past innovation intensity of other firms influences a firm's current decision on creative destruction, define the expected value of creative destruction intensity in period 1 as $\overline{x}_1^* = x_{1,1}^*\big|_{q_{1,1}=q_{1,0}}\frac{1}{2}(1-z_{1,0})+x_{2,1}^*\big|_{q_{2,1}=q_{2,0}}\frac{1}{2}(1-x_{2,0})+x_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}}\frac{1}{2}z_{1,0}+x_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}}\frac{1}{2}x_{2,0}$. Then, the first part of proposition C.2 implies the following:

Corollary C.2 (Technological Barrier Effect)**.** *Given technology $q_{j,1}$ and period 0 innovation intensities $z_{1,0}$ and $x_{2,0}$, $\frac{\partial \overline{x}_1^*}{\partial z_{1,0}} < 0$ and $\frac{\partial \overline{x}_1^*}{\partial x_{2,0}} < 0$ hold.*

Proof. $\frac{\partial \overline{x}_1^*}{\partial z_{1,0}} = \frac{1}{2}\left(x_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} - x_{1,1}^*\big|_{q_{1,1}=q_{1,0}}\right) < 0$, and $\frac{\partial \overline{x}_1^*}{\partial x_{2,0}} = \frac{1}{2}\left(x_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} - x_{2,1}^*\big|_{q_{2,1}=q_{2,0}}\right) < 0$, where the signs follow from proposition C.2 □

Corollary C.2 indicates that higher technology levels in other markets, resulting from previous innovation, act as effective technological barriers, making it challenging for outside firms to take over those product markets. This reduces firms' incentives for creative destruction. Lastly, because innovation is forward-looking, changes in future profits π' are crucial factors influencing the current period's innovation. Proposition C.3 summarizes this:

Proposition C.3 (Ex-post Schumpeterian Effect)**.** *Given the expected profit $\pi_{j,2}$ in period 2, we obtain $\frac{\partial z_{j,1}^*}{\partial \pi_{j,2}} > 0\ \forall q_{j,1}$ and $\frac{\partial x_{j,1}^*}{\partial \pi_{j,2}} > 0$ for $q_{j,1} = q_{j,0}$. The signs for $\frac{\partial x_{j,1}^*}{\partial \pi_{j,2}}$ for other technology gaps remain ambiguous.*

Proof. For $q_{j,1} = q_{j,0}$, we have $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}(\lambda-1)(1-x_{j,1})(1-x_1^e) - \frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda-1)(1-x_1^e)\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ and $\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{\eta}{2\widetilde{\chi}}$. Thus, $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}(\lambda-1)(1-2x_{j,1})(1-x_1^e) > 0$ iff $x_{j,1} < \frac{1}{2}$. For $q_{j,1} = \lambda q_{j,0}$, we get $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} > 0$ unambiguously. For $q_{j,1} = \eta q_{j,0}$, $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}\left[\lambda-\frac{1}{2}-\frac{1}{2}(1-x_{j,1})(1-x_1^e)\right]+\frac{\pi_{j,2}}{2\widehat{\chi}}\frac{1}{2}(1-x_1^e)\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ and $\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{\eta}{2\widetilde{\chi}}\frac{1}{2}(1-z_{j,1}) - \frac{\eta \pi_{j,2}}{2\widetilde{\chi}}\frac{1}{2}\frac{\partial z_{j,1}}{\partial \pi_{j,2}}$ are obtained, and we get $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \left[\lambda - \frac{1}{2} - \frac{1}{2}(1-2x_{j,1})(1-x_1^e)\right]\left[2\widehat{\chi} + \frac{\eta(\pi_{j,2})^2}{2\widetilde{\chi}}\frac{1}{4}(1-x_1^e)\right]^{-1} > 0$. The sign for $\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ remains ambiguous. □

Proposition C.3 implies that any factor that affects future profits may influence firms' own-innovation and creative destruction. Specifically, an increase in expected profit from one's own market encourages firms to intensify their own-innovation efforts. However, the impact of an increase in expected profit in other markets on firms' decisions regarding creative destruction is ambiguous when the local technology gap exceeds 1. This ambiguity arises because incumbents in these markets tend to increase their own-innovation efforts in response to higher expected profits, thereby allowing them to protect their markets. In cases where the local technology gap equals 1, incumbents cannot protect their markets through own-innovation alone. Consequently, an increase in expected future profit unambiguously stimulates creative destruction in such scenarios. These findings highlight the diverse factors influencing own-innovation, creative destruction, and overall innovation levels.

D Extension: Stochastic Innovation Step Size

In this section, we extend our baseline model by relaxing the constant innovation step size assumption. We demonstrate that the predictions of our baseline model remain robust without assuming that $\lambda^2 > \eta$. Thus, $\lambda^2 > \eta$ is an innocuous simplifying assumption serving only to clarify the exposition of the main mechanism and reduce computational burden. Following Garcia-Macia et al. (2019), we let firms draw innovation step sizes from probability distributions. Successful innovation improves product quality by a step size drawn from a distribution. For own-innovation, $\lambda \sim \widehat{\mu}(\lambda)$, where $\lambda \in [\lambda_L, \lambda_U]$ with mean $\overline{\lambda}$; for creative destruction, $\eta \sim \widetilde{\mu}(\eta)$, where $\eta \in [\eta_L, \eta_U]$ with mean $\overline{\eta}$. Here, $\lambda_L \geq 1$ and $\eta_L \geq 1$ hold. To be consistent with our empirical findings in Section 3.2.2, we assume $\overline{\eta} \geq \overline{\lambda}$. Under this setup, the technology gap is continuous, taking values $\Delta \in [1, \eta_U]$.

Innovation by Incumbents Consider firm A, which owns product 1 with quality q_{1t} and technology gap Δ_{1t}, where $q_{1t} = \Delta_{1t} q_{1t-1}$, and $\Delta_{1t} \in [1, \eta_U]$. For simplicity, assume firms exit the economy in $t+1$ after receiving profits from their products. If firm A retains product 1 in $t+1$, it receives a profit of πq_{1t+1} and zero otherwise. Furthermore, if firm A succeeds in taking over product 2 owned by firm B, it receives a profit of πq_{2t+1}. The value function of firm A in t is then $V(q_{1t}, \Delta_{1t}) = \max_{z_{1t}, x_{2t}^A} \big\{ \pi \Delta_{1t} q_{1t-1} - \widehat{\chi} z_{1t}^{\widehat{\psi}} \Delta_{1t} q_{1t-1} - \widetilde{\chi} (x_{2t}^A)^{\widetilde{\psi}} q_{2t-1} + \widetilde{\beta} \mathbb{E}_{\{\lambda_{jt}, \eta_{jt}\}_{j=1}^2} \big[(1 - z_{1t})(1 - x_{1t}^B + x_{1t}^B Pr(\Delta_{1t} \geq \eta_{1t})) \pi \Delta_{1t} q_{1t-1} + z_{1t}(1 - x_{1t}^B + x_{1t}^B Pr(\Delta_{1t} \lambda_{1t} \geq \eta_{1t})) \pi \Delta_{1t} \lambda_{1t} q_{1t-1} + x_{2t}^A (z_{2t} Pr(\eta_{2t} > \Delta_{2t} \lambda_{2t}) + (1 - z_{2t}) Pr(\eta_{2t} > \Delta_{2t})) \pi \eta_{2t} q_{2t-1} \big] \big\}$. The first three terms represent current period profits net of R&D costs for own-innovation ($\widehat{\chi} z_{1t}^{\widehat{\psi}} \Delta_{1t} q_{1t-1}$) and creative destruction ($\widetilde{\chi} (x_{2t}^A)^{\widetilde{\psi}} q_{2t-1}$). The terms inside the expectation operator correspond to the expected profits from the existing product (product 1) and that from taking over product 2 through creative destruction. Here, z_{2t}, x^B_{1t}, and Δ_{2t} represent firm B's respective counterparts for own-innovation and creative destruction intensities, and technology gap.

Taking the first-order conditions with respect to z_{1t} and x^A_{2t} yields firm A's optimal own-innovation decision $z^*_{1t} = \left(\widetilde{\beta}\pi/\widehat{\chi}\widehat{\psi}\right)^{1/(\widehat{\psi}-1)}\left[(\overline{\lambda}-1)(1-x^B_{1t}) + x^B_{1t}\mathbb{E}_{\lambda_{1t},\eta_{1t}}\{\lambda_{1t}Pr(\Delta_{1t}\lambda_{1t} \geq \eta_{1t})\} - x^B_{1t}\mathbb{E}_{\eta_{1t}}\{Pr(\Delta_{1t} \geq \eta_{1t})\}\right]^{1/(\widehat{\psi}-1)}$, and optimal creative destruction decision $x^{A*}_{2t} = \left(\widetilde{\beta}\pi/\widetilde{\chi}\widetilde{\psi}\right)^{1/(\widetilde{\psi}-1)}$ $\left[z_{2t}\mathbb{E}_{\lambda_{2t},\eta_{2t}}\{\eta_{2t}Pr(\eta_{2t} > \Delta_{2t}\lambda_{2t})\} + (1-z_{2t})\mathbb{E}_{\eta_{2t}}\{\eta_{2t}Pr(\eta_{2t} > \Delta_{2t})\}\right]^{1/(\widetilde{\psi}-1)}$. The following proposition shows that the changes in firms' own-innovation decisions in response to increasing competition mirror those in the baseline model. To prove this analytically, we assume that the two step sizes are drawn from uniform distributions, $\mathcal{U}(\cdot,\cdot)$.

Proposition D.1 (Market-Protection Effect)**.** *Suppose $\lambda \sim \mathcal{U}(\lambda_L, \lambda_U)$ and $\eta \sim \mathcal{U}(\eta_L, \eta_U)$, with $\overline{\eta} \geq \overline{\lambda}$, $\lambda_U > \eta_L$, and equal variances. Then, $\frac{\partial z^*_{1t}(x^B_{1t},\Delta_{1t})}{\partial x^B_{1t}}$ is hump-shaped with respect to the technology gap Δ_{1t} and is positive over a region that includes $[\frac{\eta_L}{\lambda_L}, \eta_U)$ when $\frac{\lambda_U}{\lambda_L} \in (1,4)$. Additionally, $\left.\frac{\partial z^*_{1t}(x^B_{1t},\Delta_{1t})}{\partial x^B_{1t}}\right|_{\Delta_{1t}=\eta_U} = 0$, while the sign of $\left.\frac{\partial z^*_{1t}(x^B_{1t},\Delta_{1t})}{\partial x^B_{1t}}\right|_{\Delta_{1t}=1}$ remains ambiguous.*

Proof. The sign of $\frac{\partial z^*_{1t}(x^B_{1t},\Delta_{1t})}{\partial x^B_{1t}}$ follows the sign of $** = -(\overline{\lambda}-1) - \mathbb{E}_{\eta_{1t}}\{Pr(\Delta_{1t} \geq \eta_{1t})\} + \mathbb{E}_{\lambda_{1t},\eta_{1t}}\{\lambda_{1t}Pr(\Delta_{1t}\lambda_{1t} \geq \eta_{1t})\}$. Depending on the value of Δ_{1t}, there are three cases to consider: Case 1, where $\Delta_{1t}\lambda_L < \eta_L$; Case 2, where $\Delta_{1t}\lambda_L \in [\eta_L, \eta_U)$; and Case 3, where $\Delta_{1t}\lambda_L = \eta_U$. Without loss of generality, we normalize $\lambda_L = 1$. In Case 1, we have $\mathbb{E}_{\eta_{1t}}\{Pr(\Delta_{1t} \geq \eta_{1t})\} = 0$, and $\mathbb{E}_{\lambda_{1t},\eta_{1t}}\{\lambda_{1t}Pr(\Delta_{1t}\lambda_{1t} \geq \eta_{1t})\} = \frac{\Delta_{1t}\lambda_U - \eta_L}{6\Delta^2_{1t}(\eta_U-\eta_L)(\lambda_U-\lambda_L)}(2\Delta^2_{1t}\lambda^2_U - \Delta_{1t}\lambda_U\eta_L - \eta^2_L)$. Then, we can show that $\frac{\partial **}{\partial \Delta_{1t}} > 0$ as $\lambda_U > \eta_L > \Delta_{1t} \geq 1$, and $\frac{\partial^2 **}{\partial \Delta^2_{1t}} > 0$. Thus, $**$ is an increasing and convex function of Δ_{1t}. Furthermore, $** \mid_{\Delta_{1t}\nearrow\eta_L/\lambda_L} > 0$ when $\frac{\lambda_U}{\lambda_L} \in (1,4)$. The sign of $\left.\frac{\partial **}{\partial \Delta_{1t}}\right|_{\Delta_{1t}=1}$ is ambiguous. In Case 2, we have $\mathbb{E}_{\eta_{1t}}\{Pr(\Delta_{1t} \geq \eta_{1t})\} = \frac{\Delta_{1t}-\eta_L}{\eta_U-\eta_L}$, and $\mathbb{E}_{\lambda_{1t},\eta_{1t}}\{\lambda_{1t}Pr(\Delta_{1t}\lambda_{1t} \geq \eta_{1t})\} = \frac{1}{2(\eta_U-\eta_L)(\lambda_U-\lambda_L)}\left[\lambda^2_U(\eta_U-\eta_L) + \lambda^2_L\eta_L - \frac{1}{3\Delta^2_{1t}}(2\Delta^3_{1t}\lambda^3_L + \eta^3_U)\right]$. Then, we can show that $\frac{\partial^2 **}{\partial \Delta^2_{1t}} < 0$, $\left.\frac{\partial **}{\partial \Delta_{1t}}\right|_{\Delta_{1t}=\eta_L/\lambda_L} > 0$, $\left.\frac{\partial **}{\partial \Delta_{1t}}\right|_{\Delta_{1t}\nearrow\eta_U} < 0$, and $**\left.\right|_{\Delta_{1t}\nearrow\eta_U} = 0$. Thus, $**$ is a concave function of Δ_{1t}, achieving a maximum at $\Delta^*_{1t} \in (\eta_L/\lambda_L, \eta_U)$. Since $**\left.\right|_{\Delta_{1t}\nearrow\eta_U} = 0$, it follows that $**\left.\right|_{\Delta_{1t}=\Delta^*_{1t}} > 0$. As in Case 1, $** \mid_{\Delta_{1t}=\eta_L/\lambda_L} > 0$ when $\frac{\lambda_U}{\lambda_L} \in (1,4)$. In Case 3, we have $\mathbb{E}_{\eta_{1t}}\{Pr(\Delta_{1t} \geq \eta_{1t})\} = 1$ and $\mathbb{E}_{\lambda_{1t},\eta_{1t}}\{\lambda_{1t}Pr(\Delta_{1t}\lambda_{1t} \geq \eta_{1t})\} = \overline{\lambda}$. Therefore, $** = 0$. □

The condition $\lambda_U/\lambda_L \in (1,4)$ implies that the average quality improvement ranges from 0% to 150%. Thus, this condition is most likely satisfied in the real application. Furthermore, $\frac{\partial z^*_{1t}(x^B_{1t},\Delta_{1t})}{\partial x^B_{1t}} > 0$ in a region near $\Delta_{1t} = \eta_U$, even without imposing this condition. Although the sign of $\frac{\partial **}{\partial \Delta_{1t}}\big|_{\Delta_{1t}=1}$ is ambiguous, numerical analysis shows that it is negative as long as η_L and λ_L is not significantly different. For example, in our baseline model calibration, we have $\overline{\lambda} = 1.04$ and $\overline{\eta} = 1.075$, which implies $\frac{\eta_L}{\lambda_L} = 1.034$, and $\frac{\lambda_U}{\lambda_L} = 1.08$. These values satisfy $\frac{\partial **}{\partial \Delta_{1t}}\big|_{\Delta_{1t}=1} < 0$.

The next proposition shows that this extended model also has the technological barrier effect.

Proposition D.2 (Technological Barrier Effect)**.** *High own-innovation intensity by an incumbent (z_{2t}) as well as a high technological barrier in the target market (Δ_{2t}) both discourage creative destruction by rival firms. Formally, $\frac{\partial x^A_{2t}(z_{2t},\Delta_{2t})}{\partial z_{2t}} < 0$, and $\frac{\partial x^A_{2t}(z_{2t},\Delta_{2t})}{\partial \Delta_{2t}} < 0$.*

Proof. Since $\lambda_{2t} \geq 1$ and $\eta_{2t} \geq 1$, we have $\mathbb{E}_{\eta_{2t}}\{\eta_{2t} Pr(\eta_{2t} > \Delta_{2t})\} > \mathbb{E}_{\eta_{2t},\lambda_{2t}}\{\eta_{2t} Pr(\eta_{2t} > \Delta_{2t}\lambda_{2t})\}\ \forall\ \Delta_{2t} \geq 1$. Thus, $\frac{\partial x^A_{2t}(z_{2t},\Delta_{2t})}{\partial z_{2t}} < 0$. Furthermore, $\mathbb{E}_{\eta_{2t},\lambda_{2t}}\{\eta_{2t} Pr(\eta_{2t} > \Delta_{2t}\lambda_{2t})\}$ is a decreasing function of Δ_{2t}. Thus, $\frac{\partial x^A_{2t}(z_{2t},\Delta_{2t})}{\partial \Delta_{2t}} < 0$. □

E Extension: Multi-Creative Destruction

We extend our baseline model by allowing firms to do multiple creative destructions. The household's problem, as well as the production decisions of the final good producer and intermediate producers, remain unchanged. We therefore focus on intermediate producers' innovation decisions and aggregate variables that are affected by the multi-creative destruction.

E.1 Optimal Innovation Decision

Following Klette and Kortum (2004) and several follow-on studies, we model firms' creative destruction decisions based on the number of products they produce (n_f). Creative destruction can be viewed as a spin-off derived from each firm's existing products. Consider product j firm f owns with quality q_j and technology gap Δ^ℓ_j. In the subsequent period, the evolution of this product can result in six cases: firm f i) loses product j and business takeover (through creative destruction) fails, ii) loses product j and takeover succeeds, iii) keeps product j while both own-innovation and takeover fail, iv) keeps product j while own-innovation fails, but takeover succeeds, v) keeps product j with successful own-innovation, but takeover fails, and vi) keeps product j with successful own-innovation and takeover. Denoting the product-technology gap pair for a product that firm f acquires through successful business takeover in the next period as $\{(q', \Delta')\}$, we can write down the evolution of the product portfolio stemming from $\Phi^f = \{(q_j, \Delta_j^\ell)\}$ for $\ell \in \{1,2,3,4\}$ for each of the six cases. For example, for $\Delta_j^\ell = \Delta^2$, $\Phi_j^{f\prime} = \emptyset \cup \emptyset$ with prob. $\overline{x}(1-z^2)(1-x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime} = \emptyset \cup \{(q', \Delta')\}$ with prob. $\overline{x}(1-z^2)(x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime} = \{(q_j, \Delta^1)\} \cup \emptyset$ with prob. $(1-\overline{x})(1-z^2)(1-x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime} = \{(q_j, \Delta^1)\} \cup \{(q', \Delta')\}$ with prob. $(1-\overline{x})(1-z^2)(x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime} = \{(\Delta^2 q_j, \Delta^2)\} \cup \emptyset$ with prob. $z^2(1-x\overline{x}_{\text{takeover}})$, and $\Phi_j^{f\prime} = \{(\Delta^2 q_j, \Delta^2)\} \cup \{(q', \Delta')\}$ with prob. $z^2(x\overline{x}_{\text{takeover}})$.[4]

If the value function is additively separable with respect to each product a firm produces, we only need to solve it at the product level and aggregate it to the firm level. For product j with $\Phi_j^f = \{(q_j, \Delta^\ell)\}$, the value function is given by $V(\Phi_j^f) = \max_{z_j, x_j} \{\pi q_j - \widehat{\chi} z_j^{\widehat{\psi}} q_j - \widetilde{\chi} x_j^{\widetilde{\psi}} \overline{q} - F\overline{q} + \widetilde{\beta}\mathbb{E}\big[V'\big(\Phi_j^{f\prime}\big)|\Phi_j^f, z_j, x_j\big]\}$, where $F\overline{q}$ represents fixed operating costs.[5] The value function for firm f with a portfolio of product quality and technology gap is then: $\Phi^f = \{\Phi_j^f\}_{j\in\mathcal{J}^f}$ is $V\big(\Phi_j^f\big) = \sum_{j\in\mathcal{J}^f} V\big(\Phi_j^f\big)$. The following proposition derives analytic expressions for firms' decision rules.[6]

Proposition E.1. *Given a technology gap distribution $\{\mu(\Delta^\ell)\}_{\ell=1}^4$, a fixed cost of operation equal to $F\overline{q} = \widetilde{\beta}B(1+g)\overline{q}$, and the exit value for a product given by $V(\emptyset) = \frac{B\overline{q}}{1-x\overline{x}_{\text{takeover}}}$, the value function of firm f with a product quality and technology gap portfolio of $\Phi^f \equiv \{(q_j, \Delta_j)\}_{j\in\mathcal{J}^f}$ is: $V(\Phi^f) = \sum_{\ell=1}^4 A_\ell\big(\sum_{j\in\mathcal{J}^f|\Delta_j=\Delta^\ell} q_j\big) + n_f B\overline{q}$, where $A_1 = \pi - \widehat{\chi}(z^1)^{\widehat{\psi}} + \widetilde{\beta}\big[A_1(1-\overline{x})(1-z^1) + \lambda A_2(1-\overline{x})z^1\big]$, $A_2 = \pi - \widehat{\chi}(z^2)^{\widehat{\psi}} + \widetilde{\beta}\big[A_1(1-\overline{x})(1-z^2) + \lambda A_2 z^2\big]$, $A_3 = \pi - \widehat{\chi}(z^3)^{\widehat{\psi}} + \widetilde{\beta}\big[A_1\big(1-\frac{1}{2}\overline{x}\big)(1-z^3) + \lambda A_2 z^3\big]$, $A_4 = \pi - \widehat{\chi}(z^4)^{\widehat{\psi}} + \widetilde{\beta}\big[A_1(1-\overline{x})(1-z^4) + \lambda A_2\big(1-\frac{1}{2}\overline{x}\big)z^4\big]$, and $B = \big[x\widetilde{\beta}A_{\text{takeover}} - \widetilde{\chi}x^{\widetilde{\psi}}\big]/\big[1-\widetilde{\beta}(1+g)x\overline{x}_{\text{takeover}}\big]$, and the optimal innovation probabilities are $z^1 = \big[\widetilde{\beta}\big[(1-\overline{x})\lambda A_2 - (1-\overline{x})A_1\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^2 = \big[\big[\widetilde{\beta}\big[\lambda A_2 - (1-\overline{x})A_1\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^3 = \big[\big[\widetilde{\beta}\big[\lambda A_2 - \big(1-\frac{1}{2}\overline{x}\big)A_1\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^4 = \big[\big[\widetilde{\beta}\big[\lambda\big(1-\frac{1}{2}\overline{x}\big)A_2 - \big(1-\overline{x}\big)A_1\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, and $x = \big[\big[\widetilde{\beta}\big[A_{\text{takeover}} + \overline{x}_{\text{takeover}}B(1+g)\big]/\big[\widetilde{\psi}\widetilde{\chi}\big]\big]^{\frac{1}{\widetilde{\psi}-1}}$, where g is the average product quality growth rate in the economy, A_{takeover} is the ex-ante value of a product line obtained from successful takeover, defined as $A_{\text{takeover}} \equiv \frac{1-z^3}{2}A_1\mu(\Delta^3) + \big(1-\frac{z^4}{2}\big)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)$, and $\overline{x}_{\text{takeover}} = \mu(\Delta^1) + (1-z^2)\mu(\Delta^2) + \frac{1}{2}(1-z^3)\mu(\Delta^3) + \big(1-\frac{1}{2}z^4\big)\mu(\Delta^4)$.*

Proof. Suppose the value function is additively separable with respect to each product a firm pro-

[4] For simplicity, we use the unconditional probability of business takeover $x\overline{x}_{\text{takeover}}$ abusively.

[5] This is commonly assumed for tractability (Akcigit and Kerr, 2018; De Ridder, 2024; Argente et al., 2024)

[6] The analytic expression for startup decisions remains unchanged.

duces. Then, we can rewrite the expected future value term for each technology gap case as follows: for Δ^1, $\mathbb{E}\big[V'\big(\Phi^{f\prime}\big)|\Phi^f, z^1, x\big] = (1-\overline{x})(1-z^1)V'\big(\big\{(q_j, \Delta^1)\big\}\big) + (1-\overline{x})z^1 V'\big(\big\{(\Delta^2 q_j, \Delta^2)\big\}\big) + x\mathbb{E}V'\big(\big\{(q', \Delta')\big\}\big) + \overline{x}(1-x\overline{x}_{\text{takeover}})V'(\emptyset)$; for Δ^2, $\mathbb{E}\big[V'\big(\Phi^{f\prime}\big)|\Phi^f, z^2, x\big] = (1-\overline{x})(1-z^2)V'\big(\big\{(q_j, \Delta^1)\big\}\big) + z^2 V'\big(\big\{(\Delta^2 q_j, \Delta^2)\big\}\big) + x\mathbb{E}V'\big(\big\{(q', \Delta')\big\}\big) + \overline{x}(1-z^2)(1-x\overline{x}_{\text{takeover}})V'(\emptyset)$, for Δ^3, $\mathbb{E}\big[V'\big(\Phi^{f\prime}\big)|\Phi^f, z^3, x\big] = \big(1-\frac{1}{2}\overline{x}\big)(1-z^3)V'\big(\big\{(q_j, \Delta^1)\big\}\big) + z^3 V'\big(\big\{(\Delta^2 q_j, \Delta^2)\big\}\big) + x\mathbb{E}V'\big(\big\{(q', \Delta')\big\}\big) + \frac{1}{2}\overline{x}(1-z^3)(1-x\overline{x}_{\text{takeover}})V'(\emptyset)$; and for Δ^4, $\mathbb{E}\big[V'\big(\Phi^{f\prime}\big)|\Phi^f, z^4, x\big] = (1-\overline{x})(1-z^4)V'\big(\big\{(q_j, \Delta^1)\big\}\big) + \big(1-\frac{1}{2}\overline{x}\big)z^4 V'\big(\big\{(\Delta^2 q_j, \Delta^2)\big\}\big) + x\mathbb{E}V'\big(\big\{(q', \Delta')\big\}\big) + \overline{x}\big(1-\frac{1}{2}z^4\big)(1-x\overline{x}_{\text{takeover}})V'(\emptyset)$.

Using the guessed value function $V(\big\{(q_j, \Delta^\ell)\big\}) = A_\ell q_j + B\overline{q}$, solving for the FONCs with respect to z^ℓ and x, and applying the suggested forms for fixed costs and the exit value, we obtain the analytic expressions for the optimal innovation decisions. For example, if $\Delta^\ell = \Delta^1$, we get $A_1 q_j + B\overline{q} = \pi q_j - \widehat{\chi} z_j^{\widehat{\psi}} q_j - \widetilde{\chi} x_j^{\widetilde{\psi}} \overline{q} + \widetilde{\beta}\Big[(1-\overline{x})(1-z^1)A_1 q_j + (1-\overline{x})z^1 A_2 \Delta^2 q_j + x_j\left[A_{\text{takeover}} + \overline{x}_{\text{takeover}}(1+g)B\right]\overline{q}\Big]$, as the fixed cost of operation and the exit value cancel out some terms associated with B. The FONC with respect to z_j is $\frac{\partial}{\partial z_j} = \widehat{\psi}\widehat{\chi} z_j^{\widehat{\psi}-1} = \widetilde{\beta}\Big[(1-\overline{x})A_2\Delta^2 - (1-\overline{x})A_1\Big]$. This equation provides the optimal own-innovation decision for Δ^1 case, which only depends on the technology gap. The FONC with respect to x_j is $\frac{\partial}{\partial x_j} = \widetilde{\psi}\widetilde{\chi} x_j^{\widetilde{\psi}-1} = \widetilde{\beta}\left[A_{\text{takeover}} + \overline{x}_{\text{takeover}}(1+g)B\right]$. This equation provides the optimal creative destruction x, which is independent of both product quality and technology gap. Collecting terms with q_j gives us the expression for A_1, which only depends on the technology gap, and collecting terms with $\overline{q}$ gives us the expression for B, which is independent of both product quality and technology gap. The remaining three technology gap cases follow the same process. These results confirm the additive separability of the value function with respect to each product-technology gap pair. □

E.2 Technology Gap Distribution Transition

From the quality evolution for incumbents (in the main text) and outsiders (Section B.2) the inflows and outflows for technology gap distribution ($\mu(\Delta^\ell)$) are defined as follows: for Δ^1, inflow is $(1-z^2)(1-\overline{x})\mu(\Delta^2) + (1-z^3)\mu(\Delta^3) + (1-z^4)(1-\overline{x})\mu(\Delta^4)$ and outflow is $(\overline{x} + z^1(1-\overline{x}))\mu(\Delta^1)$; for Δ^2, inflow is $z^1(1-\overline{x})\mu(\Delta^1) + z^3\mu(\Delta^3) + (z^4 + (1-z^4)\overline{x})\mu(\Delta^4)$ and outflow is $(1-z^2)\mu(\Delta^2)$; for Δ^3, inflow is $\overline{x}\mu(\Delta^1)$ and outflow is $\mu(\Delta^3)$; and for Δ^4, inflow is $(1-z^2)\overline{x}\mu(\Delta^2$ and outflow is $\mu(\Delta^4)$.

E.3 Aggregate Variables

Aggregate Creative Destruction Arrival Rate Firms do creative destruction for each product they own simultaneously. Given the unit mass of products, there is a unit mass of creative destruction trials by incumbent firms each period. Defining $s_d = \mathcal{F}_d/\mathcal{F}$ as the share (the total mass) of domestic products and $s_o = \mathcal{F}_o/\mathcal{F}$ as the outside counterpart, we can write the aggregate creative destruction arrival rate as $\overline{x} = s_d x + \mathcal{E}_d x_e + \underbrace{s_o x + \mathcal{E}_o}_{\equiv \overline{x}_o}$, where $\mathcal{E}_o$ is the total mass of potential outside entrants with successful creative destruction. As we assume the symmetry between domestic and outside firms, the outsiders' creative destruction intensity is also x. As $s_d + s_o = 1$, we can rewrite $\overline{x}$ as $\overline{x} = x + \mathcal{E}_d x_e + \mathcal{E}_o$.

Aggregate Productivity Growth Decomposition The total mass of domestic creative destruction trials is the share of products owned by domestic firms s_d, given the unit mass assumption. Thus, we can replace the mass of domestic firms ($\mathcal{F}_d$) with s_d and obtain the following decomposition as in the single creative destruction setup:

$$
\begin{aligned}
g = & \underbrace{\left(\Delta^2 - 1\right) s_d \left[(1-\overline{x}) z^1 \mu(\Delta^1) + z^2 \mu(\Delta^2) + z^3 \mu(\Delta^3) + \left(1 - \overline{x}/2\right) z^4 \mu(\Delta^4)\right]}_{\text{own-innovation by domestic incumbents}} \\
& + \underbrace{\left(\Delta^2 - 1\right) (1 - s_d) \left[(1-\overline{x}) z^1 \mu(\Delta^1) + z^2 \mu(\Delta^2) + z^3 \mu(\Delta^3) + \left(1 - \overline{x}/2\right) z^4 \mu(\Delta^4)\right]}_{\text{own-innovation by foreign firms}} \\
& + \underbrace{\left(\overline{\Delta^{\text{ex}}} - 1\right) s_d x \mu(\overline{\Delta^{\text{ex}}})}_{\text{creative destr. by domestic incumbents}} + \underbrace{\left(\overline{\Delta^{\text{ex}}} - 1\right) \mathcal{E}_d x_e \mu(\overline{\Delta^{\text{ex}}})}_{\text{creative destr. by domestic startups}} + \underbrace{\left(\overline{\Delta^{\text{ex}}} - 1\right) \overline{x}_o \mu(\overline{\Delta^{\text{ex}}})}_{\text{creative destr. by foreign firms}} .
\end{aligned}
$$

Aggregate Domestic R&D Expenses Similarly, the aggregate domestic R&D expenses can be rephrased as $R_d = \widehat{\chi} \sum_{\ell=1}^{4} \left[\int_0^1 q_j \mathcal{I}_{\{\Delta_j = \Delta^\ell, j \in \mathcal{D}\}} dj\right] (z^\ell)^{\widehat{\psi}} + s_d \widetilde{\chi} \overline{q} x^{\widetilde{\psi}} + \mathcal{E}_d \widetilde{\chi}_e (x_e)^{\widetilde{\psi}_e} \overline{q}$.

Aggregate Consumption Households own both final goods and domestic intermediate producers. They fund the R&D expenses of domestic potential startups and pay the exit value to domestic incumbents. The households earn labor income from final goods producer (wL), operating fixed costs from intermediate producers ($s_d F \overline{q}$), as well as profits from both producers ($\Pi = 0$ and $\sum_{j \in \mathcal{D}} \pi q_j > 0$). Intermediate producers' profits include the exit value if their product is taken over and their own creative destruction fails. Thus, the household budget constraint is $wL + s_d F \overline{q} +$

$\int j \in \mathcal{D}\{\pi q_j - F\overline{q}\} + (1 - x\overline{x}_{\text{takeover}})V(\emptyset) = C + \mathcal{E}_d \widetilde{\chi}_e (x_e)^{\widetilde{\psi}_e} \overline{q} + (1 - x\overline{x}_{\text{takeover}})V(\emptyset)$. With the final goods producers' profit function $\Pi = Y - \int_{j \in \mathcal{D}} p_j y_j dj - \int_{j \notin \mathcal{D}} p_j y_j dj - wL$, the aggregate consumption is $C = Y - \int_{j \notin \mathcal{D}} p_j y_j dj - Y_d - R_d$.

F Extension: Duopolistic Competition

This section extends the baseline model to incorporate duopolistic competition in the product market, where two firms—one frontier and one laggard—operate. As in the baseline model, firms can engage in both creative destruction and own-innovation. In the case of creative destruction, innovation is undirected, and the successful firm takes over the leadership position in the target market. In the case of two incumbent firms in a product market having the same probability to be taken over from creative destruction, either firm can be taken over with equal probability. The firm not displaced becomes the laggard, positioned one step behind the new leader. We follow the functional assumptions of Cavenaile et al. (2019) for tractability.

F.1 Final Goods Producer

The final goods producer manufactures the final good using a continuum of differentiated intermediate goods indexed by $j \in [0, 1]$ as follows:

$$Y_t = \exp\left[\frac{1}{1-\theta}\int_0^1 \ln\left((y_{jt}^f)^{1-\theta} + (y_{jt}^{-f})^{1-\theta}\right)dj\right], \tag{1}$$

where y_{jt}^f and y_{jt}^{-f} are the quantity of good j, provided by firm f and the other competitor $-f$ in the market, respectively. The market is competitive, with the price normalized to one, and producers take input prices as given.

Given this, they solve the following maximization and obtain demand function:

$$\max_{y_{jt}^f, y_{jt}^{-f}} Y - \int_0^1 (p_{jt}^f y_{jt}^f + p_{jt}^{-f} y_{jt}^{-f})dj$$

subject to (1). The first order conditions give the following demand functions and relationship:

$$p_{jt}^f = Y \frac{(y_{jt}^f)^{-\theta}}{(y_{jt}^f)^{1-\theta} + (y_{jt}^{-f})^{1-\theta}} \tag{2}$$

$$\frac{p_{jt}^f}{p_{jt}^{-f}} = \Big(\frac{y_{jt}^{-f}}{y_{jt}^f}\Big)^{\theta} \tag{3}$$

F.2 Intermediate Goods Producers

The intermediate goods producers produce and sell differentiated intermediate goods to final good producers. There are two firms, leader F and laggard L, who exert duopolistic market power in each product market. Intermediate goods are produced using final goods as input at a constant unit input cost. Assuming CRS production function, the profit-maximization problem is given by:

$$\pi_{jt}^f = \max_{y_{jt}^f} p_{jt}^f y_{jt}^f - \frac{y_{jt}^f}{q_{jt}^f} \tag{4}$$

subject to (2). q_{jt}^f is the quality of the product produced by firm f.

The first-order condition gives

$$y_{jt}^f = q_{jt}^f \frac{(1-\theta)Y_t(y_{jt}^{-f}/y_{jt}^f)^{1-\theta}}{(1+(y_{jt}^{-f}/y_{jt}^f)^{1-\theta})^2}, \tag{5}$$

and given its symmetry, and using (3), we have the following relationship:

$$\frac{y_{jt}^f}{y_{jt}^{-f}} = \frac{q_{jt}^f}{q_{jt}^{-f}} = \Big(\frac{p_{jt}^f}{p_{jt}^{-f}}\Big)^{-1/\theta}. \tag{6}$$

Let $\lambda^{\hat{\Delta}jt^f} \equiv \frac{qjt^f}{q_{jt}^{-f}}$ define the quality gap of firm f relative to the other firm, where $\hat{\Delta}_{jt}^f$ represents the gap in its innovation rungs. By combining (2), (5), and (6), the profit becomes a time-invariant function of the quality gap between the two firms:

$$\pi_{jt}^f = \frac{1+\theta(q_{jt}^{-f}/q_{jt}^f)^{1-\theta}}{(1+(q_{jt}^{-f}/q_{jt}^f)^{1-\theta})^2} Y_t = \frac{1+\theta\lambda^{-\hat{\Delta}_{jt}^f(1-\theta)}}{(1+\lambda^{-\hat{\Delta}_{jt}^f(1-\theta)})^2} Y_t \equiv \pi(\hat{\Delta}_{jt}^f) Y_t. \tag{7}$$

F.3 Innovation

As in the baseline, firms can do own-innovation and creative destruction. If doing own-innovation, to produce innovation intensity z_{jt}, firms incur the following R&D cost: $R_{jt}^{own} = z_{jt}^{\hat{\psi}} \hat{\chi} Y_t$, and if doing creative destruction, firms need to pay the following R&D cost for (per-product) innovation intensity x_t: $R_{jt}^{cd} = x_t^{\tilde{\psi}} \tilde{\chi} Y_t n_t$.

F.4 Value Function

The state variables for the firm are the number of products (n_t^f), the quality gap ($\hat{\Delta}_{jt}^f$), and the quality gap from the previous period ($\Delta_{jt}^f = \frac{q_{jt}^f}{q_{jt-1}^f}$). Given that, the firm's value function can be characterized as follows:

$$\begin{aligned} V(n_t^f, \{\hat{\Delta}_{jt}^f, \Delta_{jt}^f\}_{j \in \mathcal{J}_t^f}) = \max_{\{z_{jt}^f\}_{j \in \mathcal{J}_t^f}, x_t} \sum_{j \in \mathcal{J}_t^f} \Big(\pi(\hat{\Delta}_{jt}^f) - (z_{jt}^f)^{\hat{\psi}} \hat{\chi}\Big) Y_t - x_t^{\tilde{\psi}} \tilde{\chi} Y_t n_t \\ + \beta \mathbb{E} V(n_{t+1}^f, \{\hat{\Delta}_{jt+1}^f, \Delta_{jt+1}^f\}_{j \in \mathcal{J}_{t+1}^f}). \end{aligned} \tag{8}$$

As before, we can guess the value function by decomposing it into components associated with own-innovation and those associated with creative destruction as follows:

$$V(n_t^f, \{\hat{\Delta}_{jt}^f, \Delta_{jt}^f\}_{j \in \mathcal{J}_t^f}) = \sum_{l=1}^{4} \sum_{j \in \mathcal{J}_t^f | \Delta_{jt}^f = \Delta^l} A_l(\hat{\Delta}_{jt}^f) Y_t + B n_t Y_t, \tag{9}$$

where $\Delta^1 = 1$, $\Delta^2 = \lambda$, $\Delta^3 = \eta$, and $\Delta^4 = \frac{\eta}{\lambda}$. Rephrasing (8) with (9), we have:

$$\begin{aligned} \sum_{l=1}^{4} \sum_{j \in \mathcal{J}_t^f | \Delta_{jt}^f = \Delta^l} A_l(\hat{\Delta}_{jt}^f) Y_t + B n_t Y_t = \max_{\{z_{jt}^f\}_{j \in \mathcal{J}_t^f}, x_t} \sum_{j \in \mathcal{J}_t^f} \Big(\pi(\hat{\Delta}_{jt}^f) - (z_{jt}^f)^{\hat{\psi}} \hat{\chi}\Big) Y_t - x_t^{\tilde{\psi}} \tilde{\chi} Y_t n_t \\ + \beta \sum_{l=1}^{4} \sum_{k=1}^{4} \int_{\hat{\Delta}_{jt+1}^f} \Big(P(\hat{\Delta}_{jt+1}^f, \Delta^k | \hat{\Delta}_{jt}^f, \Delta^l) \sum_{\substack{j \in \mathcal{J}_{t+1}^f | \Delta_{jt}^f = \Delta^l, \\ \Delta_{jt+1}^f = \Delta^k}} A_k(\hat{\Delta}_{jt+1}^f) Y_{t+1} + B n_{t+1} Y_{t+1}\Big) d\hat{\Delta}_{jt+1}^f, \end{aligned}$$

where $P(\hat{\Delta}_{jt+1}^f, \Delta^k | \hat{\Delta}_{jt}^f, \Delta^l)$ is the probability of switching from one state ($\hat{\Delta}_{jt}^f, \Delta_{jt}^f = \Delta^l$) to another $(\hat{\Delta}^f_{jt+1}, \Delta_{jt+1} = \Delta^k)$. Note that this can be rewritten with the product-level value function $v(\hat{\Delta}^f_{jt}, \Delta^l)$ as follows:

$$V(n^f_t, \{\hat{\Delta}^f_{jt}, \Delta^f_{jt}\}_{j\in\mathcal{J}^f_t}) = \sum_{l=1}^{4} \sum_{j\in\mathcal{J}^f_t|\Delta^f_{jt}=\Delta^l} \Big(A_l(\hat{\Delta}^f_{jt})Y_t + BY_t\Big) = \sum_{l=1}^{4} \sum_{j\in\mathcal{J}^f_t|\Delta^f_{jt}=\Delta^l} v(\hat{\Delta}^f_{jt}, \Delta^l),$$

$$\text{where } v(\hat{\Delta}^f_{jt}, \Delta^l) = \max_{z^l(\hat{\Delta}^f_{jt}),x_t} \Big(\pi(\hat{\Delta}^f_{jt}) - (z^l(\hat{\Delta}^f_{jt}))^{\hat{\psi}}\hat{\chi}\Big)Y_t - x_t^{\tilde{\psi}}\tilde{\chi}Y_t$$
$$+ \beta \sum_{k=1}^{4} \int_{\hat{\Delta}^f_{jt+1}} \Big(P(\hat{\Delta}^f_{jt+1}, \Delta^k|\hat{\Delta}^f_{jt}, \Delta^l) \sum_{\substack{j\in\mathcal{J}^f_{t+1}|\Delta^f_{jt}=\Delta^l,\\ \Delta^f_{jt+1}=\Delta^k}} A_k(\hat{\Delta}^f_{jt+1})Y_{t+1} + BY_{t+1}\Big)d\hat{\Delta}^f_{jt+1}.$$

F.5 Optimal Innovation Decision

We can solve for the optimal innovation by considering the cases where firm f is the frontier, leveled (neck-and-neck), or the laggard. For the sake of brevity, we present the frontier's decision rules in this manuscript, with the decision rules for the neck-and-neck and laggard cases available upon request.

Suppose that in product market j, firm f is the frontier with the gaps $\hat{\Delta}^f_{jt}$ and Δ^ℓ_j. As before, the evolution of this product in the subsequent period depends on several contingencies, depending on the firm's success of own-innovation and potential takeovers as well as the laggard's success of own-innovation. Note that with the duopoly setup, the laggard's innovation now also affects the firm's expected value of innovation, as it influences the quality gap and profit (market share) in the market.

Let z^F and z^L be the innovation intensity of the frontier and the laggard in a product market. Denoting the product-technology gap pair for a product that firm f acquires through successful business takeover in the next period as $\{(1, \Delta')\}$, we can write down the evolution of the product portfolio stemming from $\Phi^f = \{(\hat{\Delta}_j, \Delta^\ell_j)\}$ for $\ell \in \{1, 2, 3, 4\}$ for each possible case.[7] For example, for $\Delta^\ell_j = \Delta^2$, $\Phi^{f\prime}_j = \emptyset \cup \emptyset$ with prob. $\overline{x}(1 - z^{F2})(1 - x\overline{x}_{\text{takeover}})$, $\Phi^{f\prime}_j = \emptyset \cup \{(1, \Delta')\}$ with prob. $\overline{x}(1 - z^{F2})(x\overline{x}_{\text{takeover}})$, $\Phi^{f\prime}_j = \{(\hat{\Delta}, \Delta^1)\} \cup \emptyset$ with prob. $(1 - \overline{x})(1 - z^{F2})(1 - z^L)(1 - x\overline{x}_{\text{takeover}})$, $\Phi^{f\prime}_j = \{(\hat{\Delta}, \Delta^1)\}\cup\{(1, \Delta')\}$ with prob. $(1-\overline{x})(1-z^{F2})(1-z^L)(x\overline{x}_{\text{takeover}})$, $\Phi^{f\prime}_j = \{(\hat{\Delta}-1, \Delta^1)\}\cup\emptyset$

[7] Note that successful creative destruction will result in a quality gap corresponding to a step size from the laggard in the market.

with prob. $(1-\bar{x})(1-z^{F2})z^L(1-x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime}=\{(\hat{\Delta}-1,\Delta^1)\}\cup\{(1,\Delta')\}$ with prob. $(1-\bar{x})(1-z^{F2})z^L(x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime}=\{(\hat{\Delta}+1,\Delta^2)\}\cup\emptyset$ with prob. $(z^{F2})(1-z^L)(1-x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime}=\{(\hat{\Delta}+1,\Delta^2)\}\cup\{(1,\Delta')\}$ with prob. $(z^{F2})(1-z^L)(x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime}=\{(\hat{\Delta},\Delta^2)\}\cup\emptyset$ with prob. $(z^{F2})(z^L)(1-x\overline{x}_{\text{takeover}})$, and $\Phi_j^{f\prime}=\{(\hat{\Delta},\Delta^2)\}\cup\{(1,\Delta')\}$ with prob. $(z^{F2})(z^L)(x\overline{x}_{\text{takeover}})$.

Following this step for other cases of $\Delta_j^l=\Delta^1,\Delta^3,\Delta^4$, the following analytic expressions for for the frontier's decision rules.

Proposition F.1. *Given a technology gap distribution $\{\mu(\Delta^\ell)\}_{\ell=1}^4$, the laggard innovation intensity z^L, and the exit value for exiting product given by $V^{exit}=\beta B(1+g)Y$, the value function of firm f with a technology gap portfolio of $\Phi^f\equiv\{(\hat{\Delta}_j,\Delta_j)\}_{j\in\mathcal{J}^f}$ is: $V(\Phi^f)=\sum_{\ell=1}^4\sum_{j\in\mathcal{J}^f|\Delta_j=\Delta^\ell}A_\ell(\hat{\Delta}_j)Y+n_fBY$, where $A_1(\hat{\Delta}_j)=\pi(\hat{\Delta}_j)-\widehat{\chi}(z^{F1})^{\widehat{\psi}}+\beta(1+g)\Big[(1-\bar{x})\Big((1-z^{F1})(1-z^L)A_1(\hat{\Delta}_j)+(1-z^{F1})z^LA_1(\hat{\Delta}_j-1)+z^{F1}(1-z^L)A_2(\hat{\Delta}_j+1)+z^{F1}z^LA_2(\hat{\Delta}_j)\Big)\Big]$, $A_2(\hat{\Delta}_j)=\pi(\hat{\Delta}_j)-\widehat{\chi}(z^{F2})^{\widehat{\psi}}+\beta(1+g)\Big[(1-\bar{x})\Big((1-z^{F2})(1-z^L)A_1(\hat{\Delta}_j)+(1-z^{F2})z^LA_1(\hat{\Delta}_j-1)\Big)+z^{F2}(1-z^L)A_2(\hat{\Delta}_j+1)+z^{F2}z^LA_2(\hat{\Delta}_j)\Big]$, $A_3(\hat{\Delta}_j)=\pi(\hat{\Delta}_j)-\widehat{\chi}(z^{F3})^{\widehat{\psi}}+\beta(1+g)\Big[(1-\frac{1}{2}\bar{x})\Big((1-z^{F3})(1-z^L)A_1(\hat{\Delta}_j)+(1-z^{F3})z^LA_1(\hat{\Delta}_j-1)\Big)+z^{F3}(1-z^L)A_2(\hat{\Delta}_j+1)+z^{F3}z^LA_2(\hat{\Delta}_j)\Big]$, $A_4(\hat{\Delta}_j)=\pi(\hat{\Delta}_j)-\widehat{\chi}(z^{F4})^{\widehat{\psi}}+\beta(1+g)\big[(1-\bar{x})((1-z^{F4})(1-z^L)A_1(\hat{\Delta}_j)+(1-z^{F4})z^LA_1(\hat{\Delta}_j-1)\Big)+(1-\frac{1}{2}\bar{x})\Big(z^{F4}(1-z^L)A_2(\hat{\Delta}_j+1)+z^{F4}z^LA_2(\hat{\Delta}_j)\Big)\big]$, and $B=\big[x\beta(1+g)A_{\text{takeover}}-\widetilde{\chi}x^{\widetilde{\psi}}\big]/\big[1-\widetilde{\beta}(1+g)(1+x\overline{x}_{\text{takeover}})\big]$, and the optimal innovation probabilities are $z^{F1}(\hat{\Delta}_j)=\big[\beta(1+g)(1-\overline{x})\big[(1-z^L)(A_2(\hat{\Delta}_j+1)-A_1(\hat{\Delta}_j))+z^L(A_2(\hat{\Delta}_j)-A_1(\hat{\Delta}_j-1))\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^{F2}(\hat{\Delta}_j)=\big[\beta(1+g)\big((1-z^L)(A_2(\hat{\Delta}_j+1)-(1-\bar{x})A_1(\hat{\Delta}_j))+z^L(A_2(\hat{\Delta}_j)-(1-\bar{x})A_1(\hat{\Delta}_j-1))\big)\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^{F3}(\hat{\Delta}_j)=\big[\beta(1+g)\big((1-z^L)(A_2(\hat{\Delta}_j+1)-(1-\frac{1}{2}\bar{x})A_1(\hat{\Delta}_j))+z^L(A_2(\hat{\Delta}_j)-(1-\frac{1}{2}\bar{x})A_1(\hat{\Delta}_j-1))\big)\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^{F4}(\hat{\Delta}_j)=\big[\beta(1+g)\big((1-z^L)((1-\frac{1}{2}\bar{x})A_2(\hat{\Delta}_j+1)-(1-\bar{x})A_1(\hat{\Delta}_j))+z^L((1-\frac{1}{2}\bar{x})A_2(\hat{\Delta}_j)-(1-\bar{x})A_1(\hat{\Delta}_j-1))\big)\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, and $x=\big[(1+g)\beta\big[A_{\text{takeover}}+\overline{x}_{\text{takeover}}B\big]/\big[\widetilde{\psi}\widetilde{\chi}\big]\big]^{\frac{1}{\widetilde{\psi}-1}}$, where g is the average product quality growth rate in the economy, A_{takeover} is the ex-ante value of a product line obtained from successful takeover, defined as $A_{\text{takeover}}\equiv\frac{1-z^3}{2}A_1\mu(\Delta^3)+\big(1-\frac{z^4}{2}\big)A_2\lambda\mu(\Delta^4)+A_3\eta\mu(\Delta^1)+(1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)$, and $\overline{x}_{\text{takeover}}=\mu(\Delta^1)+(1-z^2)\mu(\Delta^2)+\frac{1}{2}(1-z^3)\mu(\Delta^3)+\big(1-\frac{1}{2}z^4\big)\mu(\Delta^4)$.*

Proof. Using the guessed value function and solving for the FONCs with respect to z^ℓ and x, and applying the the exit value, we obtain the analytic expressions for the optimal innovation decisions. For example, if $\Delta^\ell=\Delta^1$, we get $A_1(\hat{\Delta}_j)Y+BY=\pi(\hat{\Delta}_j)Y-\widehat{\chi}z_j^{\widehat{\psi}}Y-\widetilde{\chi}x_j^{\widetilde{\psi}}Y+\beta(1+g)\Big[\Big\{(1-$

$\bar{x})\big((1-z_j)(1-z^L)A_1(\hat{\Delta}_j)+(1-z_j)z^L A_1(\hat{\Delta}_j-1)+z_j(1-z^L)A_2(\hat{\Delta}_j+1)+z_j z^L A_2(\hat{\Delta}_j)\big)\Big\}+$ $x_j A^{takeover}+(1+x_j\bar{x}^{takeover})B\Big]Y$, as the exit value cancel out some terms associated with B. The FONC with respect to z_j is $\frac{\partial}{\partial z_j}=\widehat{\psi}\widehat{\chi}z_j^{\widehat{\psi}-1}=\beta(1+g)(1-\overline{x})\big[(1-z^L)(A_2(\hat{\Delta}_j+1)-A_1(\hat{\Delta}_j))+z^L(A_2(\hat{\Delta}_j)-A_1(\hat{\Delta}_j-1))\big]$. This equation provides the optimal own-innovation decision for Δ^1 case, which depends on the technology gap $(\hat{\Delta}_j,\Delta_j)$ and the laggard's innovation z^L. The FONC with respect to x_j is $\frac{\partial}{\partial x_j}=\widetilde{\psi}\widetilde{\chi}x_j^{\widetilde{\psi}-1}=\beta(1+g)\left[A_{\text{takeover}}+\overline{x}_{\text{takeover}}B\right]$. This equation provides the optimal creative destruction x, which is independent of the technology gaps and the laggard innovation in the market. Collecting terms associated with own-innovation gives us the expression for $A_1(\hat{\Delta}_j)$, which depends on technology gap and the laggard innovation, and collecting terms associated with creative destruction gives us the expression for B, which is independent of both technology gap and the laggard innovation. The remaining three technology gap cases for Δ_j follow the same process. These results confirm the additive separability of the value function with respect to each product-technology gap pair. □

Given the proposition, we can replicate the main results in the baseline model. First, the following corollary shows that own-innovation increases with the technology gap, but beyond a certain point, a wider technology gap can discourage further investment in own-innovation. This replicates Corollary 1 in the main text.

Corollary F.1. *In an equilibrium where $\{z^{F\ell}\}_{\ell=1}^4$ are well defined, the probabilities of own-innovation for the frontier firm satisfy $z^{F2}>z^{F3}>z^{F4}>z^{F1}$ given any levels of $\hat{\Delta}_j$ and z^L.*

Proof. Given $\hat{\Delta}_j$ and z^L, comparing z^{F1} and z^{F2} gives $(z^{F1})^{\hat{\psi}-1}-(z^{F2})^{\hat{\psi}-1}=\frac{\beta(1+g)}{\hat{\psi}\hat{\chi}}\bar{x}((1-z^L)A_2(\hat{\Delta}_j+1)+z^L A_1(\hat{\Delta}_j))>0$. Similarly, comparing z^{F2} and z^{F3}, we have $(z^{F2})^{\hat{\psi}-1}-(z^{F3})^{\hat{\psi}-1}=\frac{\beta(1+g)}{\hat{\psi}\hat{\chi}}\frac{1}{2}\bar{x}((1-z^L)A_1(\hat{\Delta}_j)+z^L A_1(\hat{\Delta}_j-1))>0$. Comparing z^{F1} and z^{F4} we obtain $(z^{F4})^{\hat{\psi}-1}-(z^{F1})^{\hat{\psi}-1}=\frac{\beta(1+g)}{\hat{\psi}\hat{\chi}}\frac{1}{2}\bar{x}((1-z^L)A_2(\hat{\Delta}_j+1)+z^L A_2(\hat{\Delta}_j))>0$. Furthermore, comparing z^{F3} and z^{F4}, we get $(z^{F3})^{\hat{\psi}-1}-(z^{F4})^{\hat{\psi}-1}=\frac{\beta(1+g)}{\hat{\psi}\hat{\chi}}\frac{1}{2}\bar{x}\Big((1-z^L)(A_2(\hat{\Delta}_j+1)-A_1(\hat{\Delta}_j))+z^L(A_2(\hat{\Delta}_j)-A_1(\hat{\Delta}_j-1))\Big)>0$. This follows $z^{F1}>0$, where $(1-z^L)(A_2(\hat{\Delta}_j+1)-A_1(\hat{\Delta}_j))+z^L(A_2(\hat{\Delta}_j)-A_1(\hat{\Delta}_j-1))>0$ needs to hold. Combining them all with $\hat{\psi}>1$ completes the proof. □

Furthermore, we can derive market-protection effect in Corollary 2 as before.

Corollary F.2 (Market-Protection Effect of Frontier)**.** *With* $\widetilde{\psi} \in (1, 2]$, *the market-protection effect has the following sign, given any levels of* $\hat{\Delta}_j$ *and* z^L*:*

$$\left.\frac{\partial z^{F2}}{\partial \bar{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^{F3}}{\partial \bar{x}}\right|_{A_1,A_2} > 0, \quad \left.\frac{\partial z^{F3}}{\partial \bar{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^{F4}}{\partial \bar{x}}\right|_{A_1,A_2} \lesseqgtr 0, \quad \textit{and} \quad 0 > \left.\frac{\partial z^{F1}}{\partial \bar{x}}\right|_{A_1,A_2}.$$

Proof. Getting the derivatives of z^{Fl} with respect to $\bar{x}$ for each $l = 1, 2, 3, 4$, given $\hat{\Delta}_j$ and z^L, we get the following signs: $\frac{\partial z^{F1}}{\partial \bar{x}} = -\left(\frac{1}{\hat{\psi}\hat{\chi}}\right)^{\frac{1}{\hat{\psi}-1}} \frac{1}{\hat{\psi}-1}(z^{F1})^{2-\hat{\psi}}\beta(1+g)((1-z^L)(A_2(\hat{\Delta}_j+1) - A_1(\hat{\Delta}_j)) + z^L(A_2(\hat{\Delta}_j) - A_1(\hat{\Delta}_j - 1))) < 0$, $\frac{\partial z^{F2}}{\partial \bar{x}} = \left(\frac{1}{\hat{\psi}\hat{\chi}}\right)^{\frac{1}{\hat{\psi}-1}} \frac{1}{\hat{\psi}-1}(z^{F2})^{2-\hat{\psi}}\beta(1+g)((1-z^L)A_1(\hat{\Delta}_j) + z^L A_1(\hat{\Delta}_j - 1)) > 0$, $\frac{\partial z^{F3}}{\partial \bar{x}} = \left(\frac{1}{\hat{\psi}\hat{\chi}}\right)^{\frac{1}{\hat{\psi}-1}} \frac{1}{\hat{\psi}-1}(z^{F3})^{2-\hat{\psi}}\beta(1+g)\frac{1}{2}((1-z^L)A_1(\hat{\Delta}_j) + z^L A_1(\hat{\Delta}_j - 1)) > 0$, $\frac{\partial z^{F4}}{\partial \bar{x}} = \left(\frac{1}{\hat{\psi}\hat{\chi}}\right)^{\frac{1}{\hat{\psi}-1}} \frac{1}{\hat{\psi}-1}(z^{F4})^{2-\hat{\psi}}\beta(1+g)((1-z^L)(A_1(\hat{\Delta}_j) - \frac{1}{2}A_2(\hat{\Delta}_j+1)) + z^L(A_1(\hat{\Delta}_j - 1) - \frac{1}{2}A_2(\hat{\Delta}_j)))$ is ambiguous. Furthermore, we can derive $\frac{\partial z^{F2}}{\partial \bar{x}} > \frac{\partial z^{F3}}{\partial \bar{x}}$ using that $z^{F2} > z^{F3}$ and $z^{F1} > 0$. This completes the proof. ☐

In addition, the following corollary shows how the frontier's innovation and market-protection effect depends on the quality gap from the laggard, $\hat{\Delta}_j$.

Corollary F.3. *In an equilibrium where* $\{z^{F\ell}\}_{\ell=1}^{4}$ *are well defined, the effect of quality gap from the laggard (or market share) on the probabilities of own-innovation for the frontier firm is ambiguous given* Δ_j.

Proof. Taking the derivatives of own-innovation with respect to the quality gap from the laggard, holding z^L fixed for simplicity, we obtain the following expressions: $\frac{\partial z^{F1}}{\partial \hat{\Delta}_j} = \frac{1}{(\hat{\psi}-1)\widehat{\psi\chi}}(z^{F1})^{2-\hat{\psi}}\beta(1+g)(1-\bar{x})\big((1-z^L)(A_2'(\hat{\Delta}_j+1) - A_1'(\hat{\Delta}_j)) + z^L(A_2'(\hat{\Delta}_j) - A_1'(\hat{\Delta}_j - 1))\big)$, $\frac{\partial z^{F2}}{\partial \hat{\Delta}_j} = \frac{1}{(\hat{\psi}-1)\widehat{\psi\chi}}(z^{F2})^{2-\hat{\psi}}\beta(1+g)\big((1-z^L)(A_2'(\hat{\Delta}_j+1) - (1-\bar{x})A_1'(\hat{\Delta}_j)) + z^L(A_2'(\hat{\Delta}_j) - (1-\bar{x})A_1'(\hat{\Delta}_j - 1))\big)$, $\frac{\partial z^{F3}}{\partial \hat{\Delta}_j} = \frac{1}{(\hat{\psi}-1)\widehat{\psi\chi}}(z^{F3})^{2-\hat{\psi}}\big[\beta(1+g)\big((1-z^L)(A_2'(\hat{\Delta}_j+1) - (1-\frac{1}{2}\bar{x})A_1'(\hat{\Delta}_j)) + z^L(A_2'(\hat{\Delta}_j) - (1-\frac{1}{2}\bar{x})A_1'(\hat{\Delta}_j - 1))\big)\big]$, $\frac{\partial z^{F4}}{\partial \hat{\Delta}_j} = \frac{1}{(\hat{\psi}-1)\widehat{\psi\chi}}(z^{F4})^{2-\hat{\psi}}\big[\beta(1+g)\big((1-z^L)((1-\frac{1}{2}\bar{x})A_2'(\hat{\Delta}_j+1) - (1-\bar{x})A_1'(\hat{\Delta}_j)) + z^L((1-\frac{1}{2}\bar{x})A_2'(\hat{\Delta}_j) - (1-\bar{x})A_1'(\hat{\Delta}_j - 1))\big)\big]$. The signs of these terms are ambiguous, depending on multiple terms associated with $A_1'(\hat{\Delta}_j)$, $A_2'(\hat{\Delta}_j)$, and $\bar{x}$. ☐

The ambiguous effect arises from the decreasing returns to scale in the profit function and the structure of R&D costs. As $\hat{\Delta}_j$ increases, marginal returns to profit diminish, which reduces firms' incentives to further enhance own-innovation beyond a certain point, combined with rising R&D costs. This leads to the observed ambiguity in the results.

However, the firm's market share has an unambiguous impact on its market protection motive, as demonstrated in the following corollary.

Corollary F.4. *If $A_1(\Delta_j)$ is an increasing function of Δ_j^F, the market protection effect for the frontier (with technology gap Δ^2 or Δ^3) becomes more pronounced as the gap Δ_j^F increases.*

Proof. Getting the derivative of $\frac{\partial z^{F2}}{\partial \bar{x}}$ and $\frac{\partial z^{F2}}{\partial \bar{x}}$ with respect to $\hat{\Delta}_j$, we can obtain that $\frac{\partial^2 z_2^F}{\partial \Delta_{jt}^F \partial \bar{x}} > 0$ and $\frac{\partial^2 z_3^F}{\partial \Delta_{jt}^F \partial \bar{x}} > 0$. This implies the amplification of the market-protection effect given $\frac{\partial z^{F2}}{\partial \bar{x}} > \frac{\partial z^{F3}}{\partial \bar{x}} > 0$. □

This result is intuitive because a leader with a higher market share will have greater incentives to foster innovation in order to protect its position in response to increased competitive pressure in the market.

G Solution Algorithm

In the model, $\{z^\ell\}_{\ell=1}^4$ are functions of $\overline{x}$; g is a function of $\overline{x}$, $\{z^\ell\}_{\ell=1}^4$, and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; x is a function of $\overline{x}$ and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; x_e is a function of $\overline{x}$ and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; and $\overline{x}$ is a function of x, and x_e. Therefore, we can solve for an equilibrium of the model by iterating over the value for the aggregate creative destruction arrival rate $\overline{x}$.

For the extended model with multiple creative destruction: i) Guess values for $\overline{x}$, g and the technology gap distribution $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; ii) Using the guess of $\overline{x}$, compute $\{A_\ell\}_{\ell=1}^4$, and $\{z^\ell\}_{\ell=1}^4$; iii) Using the guess of $\overline{x}$, g, and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$, compute B, x, x_e. Next, compute the stationary $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$, based on the guess of $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$, innovation decision rules, and the following law of motion $\mu_{n+1}(\Delta^\ell) = \mu_n(\Delta^\ell) + \mathit{inflow}_n(\Delta^\ell) - \mathit{outflow}_n(\Delta^\ell)$ for each $\ell \in \{1, 2, 3, 4\}$. Lastly, compute g_∞ with $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$; iv) Compute $\overline{x}' = x + \mathcal{E}_d x_e + \mathcal{E}_o$; v) If $\overline{x} \neq \overline{x}'$, set $\overline{x} = \overline{x}'$, $g = g_\infty$, and $\{\mu(\Delta^\ell)\}_{\ell=1}^4 = \{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$, use them as new guess, and return to ii); vi) Repeat ii) through v) until the convergence of $\overline{x}$; and vii) Simulate the model over 10,000 products for 1,200 years and compute the moments averaged across the last 150 years.

H Other Theoretical Results

Table H.1: Changes in Innovation Values

Description	Variables	Before	After	% Change
Innovation Values	A_1	0.160	0.158	-1.1%
	A_2	0.173	0.172	-1.0%
	A_3	0.182	0.180	-1.0%
	A_4	0.165	0.163	-1.1%
	B	0.011	0.011	-2.6%

Table H.2: Aggregate Growth Rate Decomposition

Description	Before	After	% Change
Average productivity growth (g, %)	2.229	2.242	0.6%
Growth by outside firms (g_o, %)	0.312	0.510	63.3%
Growth by domestic firms (g_d, %)	1.888	1.680	-11.0%
Growth from domestic own-innovation (%)	1.047	0.927	-11.4%
Growth from domestic creative destruction (%)	0.656	0.571	-13.0%
Growth from domestic startups (%)	0.186	0.182	-1.7%

Table H.3: Aggregate Growth Rate Decomposition, Holding Mass Fixed

Description	Before	After	% Change
Average productivity growth by domestic firms (%)	1.888	1.875	-0.7%
Growth from domestic own-innovation (%)	1.047	1.048	0.1%
Growth from domestic creative destruction (%)	0.656	0.645	-1.7%
Growth from domestic startups (%)	0.186	0.182	-1.7%

I Counterfactual: Competitive Pressure by Domestic Startups

We increase the mass of potential domestic startups ε_d by 15.2%, which raises the creative destruction arrival rate $\overline{x}$ from 21.5% to 21.9% (1.51% increase, equivalent to the main counterfactual exercise). Table I.1 and Panel A in Table I.2 present the results. The firm-level responses remain the same as before, while the total mass of domestic incumbents and startups increases. Thus, the moments related to the number of domestic firms and startups help identify the source behind the

Table H.4: Changes in Firm Innovation in High Creative Destruction Cost Economy

Description	Variables	Before	After	% Change
Creative destruction arrival rate by outside firms	$\overline{x}_o$	1.361	2.406	76.8%
Aggregate creative destruction arrival rate	$\overline{x}$	8.966	9.636	7.5%
Prob. of own-innovation ($\Delta^1 = 1$)	z^1	20.581	20.300	-1.4%
Prob. of own-innovation ($\Delta^2 = \lambda$)	z^2	50.357	51.024	1.3%
Prob. of own-innovation ($\Delta^3 = \eta$)	z^3	36.483	36.744	0.7%
Prob. of own-innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	35.469	35.662	0.5%
Prob. of creative destruction, incumbents	x	0.380	0.363	-4.6%
Prob. of creative destruction, potential startups	x_e	7.285	6.954	-4.6%

Table H.5: Aggregate Growth Decomposition, Low Creativity Economy, Holding Mass Fixed

Description	Before	After	% Change
Average productivity growth by domestic firms (%)	1.397	1.378	-1.4%
Growth from domestic own-innovation (%)	0.991	0.994	0.3%
Growth from domestic creative destruction (%)	0.017	0.016	-5.3%
Growth from domestic startups (%)	0.388	0.368	-5.3%

increased competitive pressure (domestic startups vs outside firms). Also, Panel B in Table I.2 displays the growth decomposition, where the aggregate growth increases (unlike the main exercise), but domestic creative destruction decreases as before.

Table I.1: Changes in Firm Innovation: Economy with More Potential Startups

Description	Variables	Before	After	% Change
Creative destruction arrival rate by outside firms	$\overline{x}_o$	3.30	3.04	-7.94%
Aggregate creative destruction arrival rate	$\overline{x}$	21.53	21.85	1.51%
Prob. of own-innovation ($\Delta^1 = 1$)	z^1	16.87	16.80	-0.42%
Prob. of own-innovation ($\Delta^2 = \lambda$)	z^2	57.83	57.95	0.20%
Prob. of own-innovation ($\Delta^3 = \eta$)	z^3	39.66	39.72	0.14%
Prob. of own-innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	37.35	37.37	0.06%
Prob. of creative destruction, incumbents	x	16.76	16.54	-1.35%
Prob. of creative destruction, potential startups	x_e	4.02	3.97	-1.35%

Table I.2: Aggregate Moment Change: Economy with More Potential Startups

Description	Before	After	% Change
Panel A: Changes in the Aggregate Moments			
Total mass of domestic firms	0.386	0.416	7.6%
Total mass of domestic startups	0.029	0.033	13.4%
R&D to sales ratio (%)	4.579	4.512	-1.5%
Avg. number of products	2.290	2.164	-5.5%
Panel B: Changes in the Aggregate Growth and Decomposition			
Average productivity growth by domestic firms (%)	1.89	1.93	2.3%
Growth from domestic own-innovation (%)	1.05	1.07	1.8%
Growth from domestic creative destruction (%)	0.66	0.65	-0.1%
Growth from domestic startups (%)	0.19	0.21	13.2%

J Data Appendix

J.1 Summary Statistics

Table J.1 and J.2 present summary statistics.

J.2 Robustness Test for Heterogeneity in Innovation

This section presents several robustness test conducted for Table 1. First, we rerun the regression using citation gaps calculated without self-citations. The results remain robust, as shown in Table J.3. Second, we test several alternative hypotheses. One possible interpretation of the current result is that firms may focus on citing expired patents to reduce the risk of infringement claims. To examine this, we restrict our sample to non-expired patents only, based on patent term information provided by the USPTO.[8] The results are presented in Table J.4, which confirm the robustness of the main findings. Additionally, we consider an alternative explanation related to the technological diversity of innovations. If an innovation spans a broader range of technologies, it may naturally cite a larger number of older patents. To address this, we control for the number of CPC classes associated with the backward-cited patents. Specifically, we include and exclude the set of technology classes linked to the focal patent itself. The results remain robust in both specifications, as shown in Table J.5 and

[8]According to the USPTO, a patent granted on a continuation, divisional, or continuation-in-part application filed on or after June 8, 1995, has a term that ends twenty years from the filing date. For patents in force on June 8, 1995, or issued on an application filed before June , 1995, the term is the greater of the twenty-year term or seventeen years from the grant date. For more details, see www.uspto.gov/web/offices/pac/mpep/s2701.html.

Table J.1: The Whole Universe of Patenting Firms vs. Regression Sample in 1992

	All patenting firms	Regression sample
Average number of patents	6.15	8.86
	(19.46)	(24.10)
Average self-citation rate	0.0434	0.0540
	(0.0899)	(0.0941)
Innovation intensity	0.055	0.093
	(0.25)	(0.33)
Number of industries operating	2.34	5.43
	(3.67)	(6.94)
Employment	511.7	1988.0
	(1869.0)	(3835.0)
Patent stock	6.45	35.22
	(26.61)	(64.37)
Employment growth	0.07	0.06
	(0.60)	(0.40)
Firm age	12.33	15.65
	(6.76)	(9.42)
7yr patent growth		-0.854
		(1.312)
7yr self-citation ratio growth		0.356
		(1.322)
Number of firms	26,500	3,100

Note: Innovation intensity in 2000 is 0.183(0.58), the seven-year patent growth in 2000 is -1.07(1.207), and the seven-year self-citation ratio growth in 2000 is 0.282(1.304).

Table J.2: Foreign Competition Shock Related Measures

	NTR gap	Dnstream NTR g.	Upstream NTR g.	NTR rate	Non-NTR r.
Mean	0.291	0.138	0.203	0.027	0.303
(Std. dev.)	(0.127)	(0.060)	(0.073)	(0.022)	(0.134)
cov(, NTR gap)		0.485	0.434	0.412	0.969
cov(, Up. NTR g.)		0.204			

J.6. Lastly, producing a novel patent may require citing a diverse set of prior patents, which could increase the likelihood of referencing very old ones. To rule this out, we control for the standard deviation of citation gaps to ensure that the average is not driven by a small number of outliers with exceptionally old application dates. The results, shown in Table J.7, confirm the robustness of our findings.

Table J.3: Backward Citation Gap and Self-Citation Ratio (No Self-citation Only)

	Citation gaps	Citation gaps	Citation gaps
Self-citation ratio	-0.276***	-0.631***	-0.781***
	(0.034)	(0.035)	(0.040)
Observations	697,968	697,968	697,968
Fixed effects	none	ct	it, ct

Note: Backward citation gap is measured without self-cited patents. Constant terms are omitted for brevity. Robust standard errors are displayed below each coefficient. The mean (standard deviation) of the backward-citation gap without self-cited patents is 7.05 (3.97). Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.4: Backward Citation Gap and Self-Citation Ratio (Non-expired Patents Only)

	Citation gaps	Citation gaps	Citation gaps
Self-citation ratio	-2.152***	-2.309***	-2.575***
	(0.018)	(0.018)	(0.020)
Observations	728,299	728,299	728,299
Fixed effects	none	ct	it, ct

Note: Backward citation gap is measured based on non-expired patents only. The expiration date of a patent is computed by the UPSTO patent term. Constant terms are omitted for brevity. Robust standard errors are displayed below each coefficient. The mean (standard deviation) of the backward-citation gap based on non-expired patents is 6.63 (3.30). Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.5: Backward Citation Gap and Self-Citation Ratio

	Citation gaps	Citation gaps	Citation gaps
Self-citation ratio	-1.837***	-2.166***	-2.386***
	(0.025)	(0.024)	(0.027)
CPC class number	0.286***	0.161***	0.121***
	(0.002)	(0.002)	(0.002)
Observations	504,607	504,607	504,607
Fixed effects	none	ct	it, ct

Note: The number of CPC classes associated with cited patents is controlled. Constant terms are omitted for brevity. Robust standard errors are displayed below each coefficient. The mean (standard deviation) of the backward-citation gap, self-citation ratio, and the number of CPC classes associated with cited patents are 7.06 (3.26), 0.12 (0.20), and 3.15 (1.97), respectively. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

J.3 Real Effect of Firm Innovation with Alternative Measures

We replicate the findings using an alternative set of measures for creative destruction and own-innovation. Specifically, creative destruction is explicitly defined by the count of patents with a zero

Table J.6: Backward Citation Gap and Self-Citation Ratio

	Citation gaps	Citation gaps	Citation gaps
Self-citation ratio	-1.823***	-2.158***	-2.379***
	(0.025)	(0.024)	(0.027)
CPC class number (excl.own)	0.292***	0.163***	0.125***
	(0.002)	(0.002)	(0.002)
Observations	504,607	504,607	504,607
Fixed effects	none	ct	it, ct

Note: The number of CPC classes associated with cited patents, excluding the focal CPC class, is controlled. Constant terms are omitted for brevity. Robust standard errors are displayed below each coefficient. The mean (standard deviation) of the backward-citation gap, self-citation ratio, and the number of CPC classes associated with cited patents are 7.06 (3.26), 0.12 (0.20), and 2.30 (1.93), respectively. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.7: Backward Citation Gap and Self-Citation Ratio

	Citation gaps	Citation gaps	Citation gaps
Self-citation ratio	-1.621***	-1.828***	-1.972***
	(0.022)	(0.021)	(0.024)
Std of citation gaps	0.623***	0.561***	0.549***
	(0.018)	(0.017)	(0.018)
Observations	670,300	670,300	670,300
Fixed effects	none	ct	it, ct

Note: The standard deviation of citation gaps is controlled. Constant terms are omitted for brevity. Robust standard errors are displayed below each coefficient. The mean (standard deviation) of the backward-citation gap and self-citation ratio are 6.87 (3.39), and 0.13 (0.21), respectively. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

self-citation ratio, while own-innovation is measured by patents with a self-citation above 0% or 10%. This more direct measure of creative destruction and own-innovation exhibits consistent and even more pronounced effects, as presented in Table J.8.

J.4 Parallel Pre-trend Assumption

We test the parallel pre-trends assumption, a key identifying assumption for the Diff-in-Diff model. We estimate (29) for the two seven-year periods preceding the policy change, 1984-1991 and 1992-1999. Table J.9 supports the validity of the assumption, where the coefficient estimates are smaller and statistically insignificant.

Table J.8: Real Effect of Innovation on Productivity Growth, Product Added, and Product Concentration (Alternative Innovation Measures)

	ΔTFPR	#prod. add	ΔHHI	ΔTFPR	#prod. add	ΔHHI
#patents (self-cite=0)	0.118**	0.358**	-0.124**	0.129**	0.354***	-0.120**
	(0.055)	(0.085)	(0.055)	(0.052)	(0.081)	(0.052)
#patents (self-cite>0.10)	-0.027	-0.274***	0.134**	-0.055	-0.317***	0.152**
	(0.053)	(0.102)	(0.063)	(0.056)	(0.118)	(0.067)
Observations	5,700	5,700	5,700	5,700	5,700	5,700
Fixed effects	jt	jt	jt	jt	jt	jt
Own-innov. cutoffs	0%	0%	0%	10%	10%	10%

Notes: Creative destruction is defined by the number of patents with a zero self-citation ratio, and own-innovation is defined by the number of patents with a self-citation above a certain cutoff. In the first three columns, the cutoff is set at zero, whereas in the last three columns, it is set at 10%. The baseline set of controls along with firm payroll, the number of operating industries and products are included. The estimates for industry (j) and the year (t) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions. Robust standard errors adjusted for clustering at the firm-level are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.9: Parallel Pre-trend Test

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap	-0.397	-0.380	-0.554	-0.546
	(0.487)	(0.488)	(0.403)	(0.402)
$\times$ Innovation intensity		-0.195		-0.058
		(0.162)		(0.395)
NTR gap $\times\ \mathcal{I}_{\{1992\}}$	0.523	0.500	0.252	0.259
	(0.355)	(0.362)	(0.294)	(0.290)
$\times$ Innovation intensity		0.092		-0.113
		(0.243)		(0.491)
Observations	5,000	5,000	5,000	5,000
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline

Notes: The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

J.5 Robustness Test for the Diff-in-Diff Identification

Furthermore, we perform several robustness checks as follows. First, we replace the baseline firm-level NTR gaps with the industry-level NTR gaps based on the primary industry (with the largest employment size) in which firms operate.[9] See Table J.10. Second, we include upstream and downstream competitive pressure shocks as covariates to control the effect of trade shocks through firms' I-O networks.[10] See Table J.11. The third test addresses a potential sampling bias using the inverse propensity score weights.[11,12] See Table J.12. The fourth test adjusts the level of standard error clustering to the firm level.[13] See Table J.13. The fifth test considers the potential correlation between the innovation intensity measure and firm size or age (e.g., Acemoglu et al., 2018), which may blur the effect of technological barriers. To address this concern, we control additional terms that interact innovation intensity with firm age and size. Moreover, we use an alternative measure based on the inverse of the innovation intensity gap relative to the industry frontier, averaged over the past five years, as the level of technological advantage. See Tables J.14 and J.15. The sixth test confirms the robustness of alternative measures for creative destruction and own-innovations. Creative destruction is directly measured by the number of new product added, and own-innovation is directly measured by the number of patents with a self-citation ratio above 0% or 10%. Also, we examine the impact on within-firm product market concentration. See Tables J.16, J.17, and J.18. Lastly, we include additional controls (such as the cumulative number of patents, firm payroll, the number of industries or products, industry-level skill and capital intensities, as well as dummies for importers and exporters) beyond the baseline set to eliminate potential alternative interpretations. See Tables J.19 and J.20.

[9]The baseline measure uses the employment-share weighted average of the industry-level NTR gaps, where the employment share is measured at the start year of each period and averaged across the firm's operating industries.

[10]The upstream (downstream) measure captures the effect of trade shocks propagating upstream (downstream) from an industry's buyers (suppliers). Using the 1992 BEA input-output table, we construct upstream and downstream competitive pressure shocks as the weighted averages of industry-level trade shocks. Following the approach in Pierce and Schott (2016), we assign I-O weights to zero for both upstream and downstream industries within the same three-digit NAICS broad industries for each six-digit NAICS industry.

[11]This issue can potentially arise from the selection of samples with a positive number of patents granted in the start year and in any of the last four years of each period in the regression analysis, which is inevitable to compute the self-citation ratio over two years for each period.

[12]To formulate the weights, we employ a logit regression on the entire universe of the LBD. The dependent variable is set to one if the firm belongs to the regression sample and zero otherwise. The independent variables include firm size, age, employment growth rate, industry, and a multi-unit status indicator.

[13]In our baseline analysis, we cluster the standard errors at the six-digit NAICS level as most variations in the firm-level NTR gap occur at the industry level.

Table J.10: Industry-level Tariff Measures

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap $\times$ Post	0.016	0.011	0.005	-0.001
	(0.249)	(0.249)	(0.261)	(0.261)
$\times$ Innovation intensity		-0.032		0.760***
		(0.229)		(0.272)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline
Weights for tariffs	major industry	major industry	major industry	major industry

Notes: Table reports results of OLS generalized difference-in-differences regressions in which industry-level tariff measures are used. The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.11: Foreign Competition Shock through I-O Linkages

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap $\times$ Post	-0.111	-0.111	-0.296	-0.424
	(0.331)	(0.342)	(0.356)	(0.355)
$\times$ Innovation intensity		-0.001		0.824***
		(0.337)		(0.288)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline+IO	baseline	baseline	baseline

Notes: The baseline set of controls is included along with the diff-in-diff terms for upstream and downstream sectors, respectively. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.12: Weighted by Inverse Propensity Score

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap × Post	0.003	0.039	-0.394	-0.603
	(0.475)	(0.484)	(0.509)	(0.512)
× Innovation intensity		-0.045		0.893***
		(0.282)		(0.294)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline
Regression weights	inv. propens.	inv. propens.	inv. propens.	inv. propens.

Notes: Table reports results of OLS generalized difference-in-differences regressions in which observations are weighted by the inverse of the propensity scores from logit model (y = indicator for analysis sample). The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. For the sake of space, only the main coefficients are presented. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.13: Standard Error Clustering on Firms

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap × Post	0.067	0.071	0.045	-0.062
	(0.287)	(0.290)	(0.308)	(0.312)
× Innovation intensity		-0.054		0.795***
		(0.245)		(0.277)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline
se. cluster	firmid	firmid	firmid	firmid

Notes: Table reports results of OLS generalized difference-in-differences regressions in which robust standard errors are adjusted for clustering at the firm-level. The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. For the sake of space, only the main coefficients are presented. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.14: Robustness Check for Innovation Intensity Measure (Firm Age, Size Effects)

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap $\times$ Post	-0.447	-0.342	0.805	0.292
	(0.645)	(0.691)	(0.668)	(0.641)
$\times$ Innovation intensity		-0.026		0.826***
		(0.239)		(0.284)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline+	baseline+	baseline+	baseline+

Notes: The baseline set of controls is included along with additional controls for the set of interaction terms between innovation intensity and firm age, as well as innovation intensity and firm size, to check robustness for potential correlations between innovation intensity, firm age, and firm size. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.15: Alternative Technology Barrier Measure

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap $\times$ Post	0.067	0.131	0.045	0.029
	(0.287)	(0.291)	(0.308)	(0.313)
$\times$ Innovation intensity		-0.058		0.066*
		(0.440)		(0.040)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline

Notes: The baseline set of controls is included, with the innovation intensity measure replaced by the past 5-year average of the inverse of the within-industry innovation intensity gap from the frontier firm as a proxy for the accumulated level of technology barriers. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.16: Alternative Creative Destruction Measure

	#products added	#products added	#products added
NTR gap $\times$ Post	-0.239***	-0.231***	-0.218***
	(0.068)	(0.067)	(0.063)
Observations	497,000	497,000	497,000
Fixed effects	j, p	j, p	j, p
Controls	baseline	baseline	baseline
Creative destruction measure	(innovation intensity)	(labor productivity)	(TFPR)

Notes: Creative destruction is directly measured by the number of products added and taken as the main dependent variable. The baseline set of controls (with a different measure for technological barriers) is included. Innovation intensity is the baseline measure as before in the first column. In the second and third columns, it is replaced by the inverse gap of the firm's labor productivity or TFPR from the frontier in its operating industry as an alternative way to measure the degree of technological barriers. Estimates for industry-period (jp) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.17: Alternative Own-Innovation Measure

	ΔPatents (self-cite>0)	ΔPatents (self-cite>0)	ΔPatents (self-cite>10)	ΔPatents (self-cite>10)
NTR gap $\times$ Post	0.007	0.001	0.005	-0.008
	(0.004)	(0.004)	(0.004)	(0.005)
$\times$ Innovation intensity		0.100***		0.206***
		(0.033)		(0.077)
Observations	497,000	497,000	497,000	497,000
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline

Notes: Own-innovation is directly measured and taken as the main dependent variable. The first two columns measure it by the number of patents with a positive self-citation ratio (self-cite > 0), and the last two columns measure it by those with at least a 10% self-citation ratio (self-cite > 10). The baseline set of controls is included. Estimates for industry-period (jp) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.18: The Effect on Product Concentration

	Δproduct HHI	Δproduct HHI
NTR gap $\times$ Post	-0.002	-0.019
	(0.042)	(0.012)
$\times$ Innovation intensity		0.262**
		(0.116)
Observations	497,000	497,000
Fixed effects	j, p	j, p
Controls	baseline	baseline

Notes: The main dependent variable is the product sales concentration within each firm. The baseline set of controls is included. Estimates for industry-period (jp) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.19: Robustness Test for the Market-Protection Effect (Overall Innovation)

	ΔPatents	ΔPatents	ΔPatents	ΔPatents	ΔPatents	ΔPatents
NTR gap $\times$ Post	0.076	0.062	0.028	0.112	0.081	0.074
	(0.283)	(0.284)	(0.284)	(0.278)	(0.279)	(0.280)
$\times$ Innov. intensity	-0.055	-0.037	-0.051	0.058	-0.055	-0.029
	(0.242)	(0.242)	(0.239)	(0.243)	(0.240)	(0.231)
Observations	6,500	6,500	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p	j, p	j, p
Controls	base+	base+	base+	base+	base+	base+

Notes: All columns augment the baseline set of controls with additional variables. Specifically, column (1) includes the cumulative number of patents, column (2) includes firm payroll, column (3) includes the number of industries in which firms operate, column (4) includes the industry-level skill, capital intensities, column (5) includes the number of industries and the industry-level skill, capital intensities, and column (6) includes the number of industries, the industry-level skill, capital intensities, a dummy for firms with total imports > 0, and a dummy for firms with total exports > 0. The estimates for industry (j) and the period (p) fixed effects, and the coefficients associated with the binary indicators are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J.20: Robustness Test for the Market-Protection Effect (Own-Innovation)

	ΔSelf-c.	ΔSelf-c.	ΔSelf-c.	ΔSelf-c.	ΔSelf-c.	ΔSelf-c.
NTR gap × Post	-0.078	-0.059	-0.026	0.007	0.042	0.063
	(0.290)	(0.291)	(0.289)	(0.287)	(0.285)	(0.285)
× Innov. intensity	0.798***	0.789***	0.792***	0.789***	0.787***	0.777***
	(0.278)	(0.278)	(0.280)	(0.277)	(0.279)	(0.268)
Observations	6,500	6,500	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p	j, p	j, p
Controls	base+	base+	base+	base+	base+	base+

Notes: All columns augment the baseline set of controls with additional variables. Specifically, column (1) includes the cumulative number of patents, column (2) includes firm payroll, column (3) includes the number of industries in which firms operate, column (4) includes the industry-level skill, capital intensities, column (5) includes the number of industries and the industry-level skill, capital intensities, and column (6) includes the number of industries, the industry-level skill, capital intensities, a dummy for firms with total imports > 0, and a dummy for firms with total exports > 0. The estimates for industry (j) and the period (p) fixed effects, and the coefficients associated with the binary indicators are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

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